The Political Economy of Underfunded Municipal Pension Plans[*]

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Abstract

This paper analyzes the determinants of underfunding of local government's pension funds using a politico-economic overlapping generations model. We show that a binding downpayment constraint in the housing market dampens capitalization of future liabilities into current land prices. Thus a local government's pension funding policy matters for land prices and the utility of young households. Underfunding arises in equilibrium if the pension funding policy is set by the old generation. Young households, instead, favor a policy of full funding. Empirical results based on cross-city comparisons in the magnitude of unfunded liabilities are consistent with the predictions of the model.

JEL Classifications: E6, H3, H7, R5

KEYWORDS: Unfunded liabilities, Political Economy, Land Prices, Capitalization.

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1 Introduction

A large number of state and local governments in the U.S. haven taken on a significant amount of debt, primarily by underfunding their public employees' pension plans. Pension plan underfunding implies that a local government incurs debt which breaks the link between current taxation and expenditure policies, allowing it to potentially shift the tax burden across cohorts. Given that cities and states face stringent requirements to balance their operating budgets each year, underfunding of pension plans is one of the few viable options to effectively take on debt that is not linked to capital expenditures. For example, Rauh and Novy-Marx (2009, 2011) have calculated that state governments unfunded liabilities amount to $3 trillion, against approximately $1 trillion of outstanding debt. According to the Pew Charitable Trust (2013), unfunded pension and health care liabilities of a sample of large U.S. cities add to several hundred billion dollars.[1] When these liabilities come due a state or a local government either needs to raise taxes or might try to renege on some of its promises. The latter option appears more difficult to implement than, for example, changing the parameters of the social security system, because local pensions are usually protected by state constitutions.[2]

Despite the potential importance of pension underfunding, we do not fully understand its political and economic determinants and its implications for the welfare of various cohorts. In this paper we investigate the politico-economic origins of local pension underfunding in the context of an analytically tractable overlapping generations model with endogenous pension

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[1] In Table A.1 in the appendix, we present Pew Charitable Trust data on liabilities and funding levels of defined benefit plans of public employees for the 20 largest U.S. cities in 2009. The unfunded portion of pension liabilities for these twenty cities alone totals $85.5 billion, with considerable variation across cities. While comprehensive funding data is not readily available for all municipalities in the U.S., this phenomenon is not confined to large urban central cities. For example, according to the 2012 Status Report on Local Government Pensions released by the Public Employee Retirement Commission of Pennsylvania, 630 out of 3,161 local pension plans in Pennsylvania were less than 80 percent funded. These estimates of unfunded liabilities are probably a lower bound as the latter are typically computed using an 8 percent discount rate following government accounting standards.

[2] The pensions of public employees of the city of Detroit were affected by this city's bankruptcy proceeding. Recent attempts by the state of Illinois of changing the negotiated pensions of public employees were, however, blocked by the state's Supreme Court (see New York Times, May 8th 2015: "Illinois Supreme Court Rejects Lawmakers' Pension Overhaul").

funding policy.[3] Agents live two periods, as young and old. Geographically mobile young agents live and work in a municipality, purchase land from old agents, and consume land services, private consumption goods and a public good. Young agents can save at the same exogenous interest rate as the local government. Our key point of departure relative to the previous literature (e.g. Conley and Rangel, 2001) is the assumption that young agents are subject to a downpayment constraint when purchasing land (housing).[4] Public goods are produced by municipal workers. The latter are compensated through a combination of wages and promised future pension benefits. The current period's policymaker in a locality chooses how much to save to finance future pension benefits, taking into account the effect of her choices on population flows, land values, and, potentially, the policies followed by next period's policymaker. The characterization of a politico-economic in our model follows the pioneering work of Krusell, Quadrini and Rios-Rull (1997) and Krusell and Rios-Rull (1999).

In overlapping generations models without altruism, Ricardian equivalence typically does not hold so that taxation and debt are not equivalent ways to finance public goods from the perspective of different generations. However, a unique and important feature of *local* debt is the so called "capitalization effect", according to which land prices reflect the mix of taxes and debt chosen by a local government. Land price capitalization has the potential to neutralize the negative welfare consequences that debt financing would otherwise produce on future generations.[5] Consider, for example, a situation in which the current policymaker reduces property taxes today, leaving a larger portion of future pensions unfunded. The reduction in property taxes increases the willingness to pay for land in the locality. However, today's land buyers anticipate that property taxes in the future will have to go up, lowering the land's

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[3]To the best of our knowledge, there are only a few papers that work out an analytical solution to Markov Perfect Equilibria of this type of dynamic political economy models. One prominent example is the work by Hassler, Mora, Storesletten and Zilibotti (2003). Other papers that analytically characterize the equilibrium of infinite horizon political economy models are Grossman and Helpman (1998), and Battaglini and Coate (2008).

[4]See, for example, Iacoviello and Pavan (2013), Campbell and Hercowitz (2005), and Favilukis, Ludvigson, and Van Nieuwerburgh (2008) for macroeconomic models with housing and downpayment constraints.

[5]The importance of land price capitalization for many issues such as debt, school quality, taxation etc. was first emphasized by Oates (1969) and has received a considerable amount of attention in the local public finance literature. See Banzhaf and Oates (2012) for a recent treatment.

future resale value. Thus, the anticipation of higher future taxes lowers the demand for land today. If young agents can freely borrow and lend at the same rate as the local government, these two effects will exactly offset each other, leaving young agents' willingness to pay for land - its user cost and, ultimately, its price - unaffected by the shift of taxes towards the future. Therefore, neither the old generation, who sells the land, nor the young one, who buys it, are affected by a local government's pension funding policy.

The distinctive feature of our model is an imperfection in the capital market. Young agents are subject to a downpayment constraint when purchasing land and can only borrow up to a fraction of their housing wealth next period. Consider now the same example as above, in which the current policymaker reduces property taxes today, leaving a larger portion of future pensions unfunded. As before, the reduction in current taxes increases young agents' willingness to pay for land on a dollar-for-dollar basis while the corresponding increase in future taxes depresses the future price of land. With a binding downpayment constraint, however, the latter effect, produces a smaller negative impact on the willingness to pay for land in the location than under perfect financial markets. The key intuition is that, since young agents are constrained, they discount changes in future land prices at a rate higher than the interest rate. As a consequence, underfunding pensions, i.e. shifting taxes to the future, increases the price young agents are willing to pay for land, benefitting the old generation of land owners. In addition, young agents' lifetime welfare is negatively affected by pension underfunding because lower current property taxes mostly benefit the old generation by raising current land values, while higher future taxes depress land prices at the time when the (then) young sell their land. It follows from the opposite preferences of young and old agents that a policy that would force a local government to increase its pension funding is bound to lead to an intergenerational conflict. The initial old generation is hurt and all subsequent generations benefit from such policy.

We test some of the model's implications using data on unfunded liabilities across 168 large U.S. cities. We find that unfunded liabilities are smaller in cities with a relative high fraction of households headed by "young" homeowners. If in these cities young households

have more political clout, this correlation is consistent with the model's prediction that young households prefer higher pension funding levels. The price elasticity of housing supply plays a key role in models of land price capitalization. Capitalization effects should occur in cities with a relatively inelastic housing supply. In the context of our model, if the price of land was exogenous, old agents would not be concerned with pension funding and young agents would favor underfunding. Based on this insight and using density as a proxy for (the inverse of) house price elasticity, we further show that the correlation between the share of young homeowners in a city and various pension underfunding measures is positive in cities with relatively low population density and negative in cities with relatively high density.

Our paper is related to a number of literatures. One is the growing literature on dynamic political-economy models in local public finance. In a related paper, Barseghyan and Coate (2015) develop a dynamic Tiebout model similar in spirit to ours and use it to study the efficiency of zoning regulations. Other contributions to this literature include Schultz and Sjostrom (2001), Conley and Rangel (2001), and Conley, Driskill and Wang (2013). They show that land price capitalization might provide the right incentives to invest in intergenerational public goods and in preventing expropriation of future generations through debt financing. The key difference between these papers and ours is the fact that young agents in our model face a downpayment constraint.[6] Bohn (2011) develops a related model of pension funding in which households face an intermediation cost of borrowing. He shows that, in the context of his model, young and middle-aged households, who are assumed to be borrowers, prefer zero pension funding, while old households are indifferent about pension funding policy. These results contrast sharply with ours. In our model young households prefer full funding of promised pensions and old households are in favor of underfunding. A number of papers on local pension funding formalize and test the notion that pension promises are a "shrouded" form of compensation to public sector employees (Glaeser and Ponzetto (2014) and Bagchi (2013)).[7] Holmes and Ohanian (2014) argue that the presence

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[6]In particular, in Proposition 5 we consider the case in which the downpayment constraint does not bind. Our results in this case are analogous to those in Conley and Rangel (2001)'s Theorem 1.

[7]Albrecht (2012) estimates the impact of states' unfunded pension liabilities on housing prices. Earlier contributions to the literature on pension funding anticipated some of the themes of the current literature

of unfunded liabilities makes a city more fragile in response to a negative income shock and argue that this mechanism can account for the recent experience of Detroit. Interestingly, they assume away land price capitalization effects because land prices in Detroit are close to their lower bound, i.e. zero.

The paper is also related to the extensive dynamic political-economy literature on debt and social security in macroeconomics. In addition to the prior references, a non-exhaustive list of recent related papers includes Bassetto and Sargent (2006), Azzimonti, Battaglini and Coate (2008), Song, Storesletten and Zilibotti (2012), among others. A distinctive feature of our paper is the presence of a land market and the related issue of capitalization of unfunded liabilities into land prices. As argued above, land market capitalization can, in principle, provide an answer to Song, Storesletten and Zilibotti (2012, page 2785)'s question: "what then prevents the current generations from passing the entire bill for current spending to future generations?" Despite the fact that in their overlapping generations (OLG) model the government taxes only young agents' labor income and Ricardian equivalence does not hold, young agents have a disciplining effect on debt. This is the case because they anticipate that increasing debt today results in lower public good expenditures when they are old. Finally, this paper is related to the macroeconomic literature that studies asset prices and portfolio choices in OLG models. Recent contributions include Glover et al (2014), who measure the welfare effects of severe recessions across various cohorts and Hur (2016) who considers a version of their model with a downpayment constraint.

The rest of the paper is organized as follows. Section 2 introduces the model economy and the definition of politico-economic equilibrium. Section 3 shows that pension funding policy matters for welfare when the downpayment constraint is binding and presents the results on the intergenerational conflict over pension funding. Section 4 discusses a number of extensions of the model. Section 5 discusses some policy implications of the model. Section 6 presents some empirical evidence consistent with the basic predictions of the model. Section

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by emphasizing - in the context of somewhat more restrictive model environments - the roles of geographic mobility (Inman, 1982), differences in borrowing rates between local governments and local residents (Mumy, 1978), and intertemporal tax smoothing considerations (Epple and Schipper, 1981) in giving rise to pension underfunding.

7 concludes. The Appendix contains the proofs of all propositions.

2   A Model of Underfunding and Capitalization

In this section we first introduce our overlapping generations model of pension funding (Section 2.1). We then consider the determinants and properties of the demand for land in this economy (Section 2.2). The latter is used to define recursively a politico-economic equilibrium for the model (Section 2.3).

2.1   Framework

The model is an overlapping generations economy of a municipality embedded in a broader economy. Ex-ante identical agents live for two periods, as young and as old. As young, agents choose whether to reside in the municipality by purchasing land there, and consume. As old, agents sell their land and consume the proceedings. The municipality is characterized by a fixed mass of land and offers a certain exogenous amount of public goods to its young residents. Public goods are produced by absentee municipal employees who receive a compensation package comprised of current wages and future pension benefits. Municipal services are financed through property taxation. While current wages to municipal employees have to be financed out of current taxes, promises of future pensions may be financed when they come due. The problem of the policy-maker in each municipality is to fund the municipal pension system.

Agents' preferences are represented by the following utility function:

$$U\left(c_{yt}, l_t, c_{ot+1}\right) = u\left(c_{yt}, l_t\right) + v\left(c_{ot+1}\right) \tag{1}$$

where c_{yt} denotes consumption of the numeraire good when young, l_t denotes the services of the land purchased by the agent, and c_{ot+1} denotes consumption when old. We make the following assumptions concerning utility.

Assumption 1 *The functions $u(c_y, l)$ and $v(c_o)$ are twice differentiable and such that: i)* $u_1(c_y, l) > 0$, $u_2(c_y, l) > 0$, $v'(c_o) > 0$; *ii)* $u_{11}(c_y, l) \leq 0$, $u_{22}(c_y, l) \leq 0$, $v''(c_o) \leq 0$, *with at least one of these inequalities being strict; iii)* $u_{12}(c_y, l) \geq 0$.

The first two sets of assumptions are standard. Higher consumption of each good increases utility and the marginal utility of consumption of each good is weakly decreasing. Condition (iii) is sufficient to guarantee that the second-order condition of the agent's optimization problem under a binding downpayment constraint is satisfied.

The quantity of the public good consumed is an exogenous constant and for simplicity of notation we do not include it in the utility function.[8] Each agent is endowed with w units of the consumption good when young and has to decide how much to consume when young and old and how much land (housing) to purchase when young.[9] An agent's budget constraint is:

$$w = c_{yt} + (1 + \tau_t) q_t l_t + \frac{b_{t+1}}{R} \tag{2}$$

$$c_{ot+1} = q_{t+1} l_t + b_{t+1} \tag{3}$$

where q_t denotes the price of land in the municipality in period t. There are two assets in this economy. In addition to land, there is also a risk-less bond. The quantity of bonds purchased (or issued) by the agent is denoted by b_{t+1}, and $R > 1$ is the exogenous gross interest rate paid by a bond.[10]

The crucial feature of our analysis is a downpayment constraint on land purchases. The importance of downpayment requirements in constraining households' housing purchases has been documented by Linneman and Wachter (1989), Zorn (1989), Jones (1989), and Haurin, Hendershott and Wachter (1997), among others. We assume that borrowing is constrained

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[8]Formally, consumption of the public good can be ignored if it enters additively in utility.

[9]We assume that there is no rental market for land.

[10]The generation that is old in $t = 0$ is assumed to have no debt or assets, or $b_0 = 0$.

to a fraction of the value of land next period:

$$-b_{t+1} \leq \kappa q_{t+1} l_t \tag{4}$$

where $0 < \kappa \leq 1$ is a parameter that indexes the size of the loan relative to the future value of the land.[11] An equivalent way to express the constraint (4) is to use equation (2) and replace b_{t+1}:

$$w - c_{yt} \geq d_t l_t, \tag{5}$$

where the downpayment per unit of land is defined as:

$$d_t \equiv (1 + \tau_t) q_t - \frac{\kappa}{R} q_{t+1}. \tag{6}$$

According to (5), agents need to self-finance the downpayment d_t, where the latter is equal to the gross-of-tax price of land in the current period minus the maximum amount a young agent is able to borrow per unit of land purchased. Notice that when $\kappa = 1$ the natural borrowing limit applies and the required downpayment coincides with the user cost of land.[12] When $\kappa = 0$ the agent needs to pay for his land acquisition entirely out of own resources.

The supply of land in the municipality is fixed at an exogenous level normalized to one. If n_t young workers live in the municipality in period t, the total demand for land in a municipality is given by $n_t l_t$. Land market equilibrium requires that:

$$n_t l_t = 1. \tag{7}$$

The government of a municipality finances the provision of a local public good. The local

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[11] The advantage of the specification in the text is that when $\kappa = 1$, equation (4) coincides with the natural borrowing limit (i.e. non negativity of consumption when old), which must prevail in order to prevent default on the debt. We have experimented with versions of the model in which the downpayment constraint depends on the current, rather than the future, price of land and obtained analogous results to those presented here.

[12] The fact that in this case the downpayment coincides with the user cost of land is due to the specification of the borrowing constraint in equation (4). See Kiyotaki and Moore (1997, p.221) for a discussion of this point in the context of a model with a downpayment constraint similar to ours.

government has committed in each period to current wage payments w^g and future pension benefits b^g. We take the vector (w^g, b^g) as given and focus on the decision to fund promised benefits. The government collects revenue $\tau_t q_t$ by taxing property values and uses it to pay the wage w^g of current public sector workers, to fund some (or none) of their promised retirement benefits b^g, and to pay for the unfunded portion of the pension benefits of last period's public sector workers. Thus, in period t a municipality's budget constraint is:

$$\tau_t q_t = w^g + \frac{f_{t+1} b^g}{R} + b^g (1 - f_t) \tag{8}$$

where f_t is fraction of pensions due in period t that is funded.[13] We assume that f_t is constrained to be between some lower bound $f_{\min} \geq 0$ and one, in which case the municipality fully funds the future pensions of its employees. We interpret the lower bound $f_{\min}$ as a policy parameter that can, in principle, be manipulated by a higher level of government.[14]

The policy decision in this economy in each period t is the mix (τ_t, f_{t+1}) of current taxes and funding of future public sector pensions. We assume that (τ_t, f_{t+1}) is chosen in each period by either the current young or the current old generation in the municipality. We consider each case separately later in the paper. The timing of events within each period is as follows. Policy is set at the beginning of the period. Then, young individuals choose whether or not to locate in the municipality, and make consumption and land demand choices. Last, the land market clears. Thus, the policymaker takes into account the effect of its choices on population flows and land values within the period. She also understands the effect of its policies on the policies chosen by future policymakers.

We express the inflow of young agents to the municipality as the following function of the indirect utility it offers, denoted by V_t:

$$n_t = P(V_t). \tag{9}$$

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[13] The initial funding level $f_0 \in [f_{\min}, 1]$ is given exogenously. It is assumed to be the same for all municipalities.

[14] We discuss the role played by $f_{\min}$ in more detail in Section 5.

The function P is assumed to be bounded, differentiable, and increasing in V_t. In what follows, we replace n_t in equation (7) using equation (9) and refer to the resulting equation:

$$P(V_t)l_t = 1, \tag{10}$$

as the land market clearing condition. The left-hand side of this equation represents the aggregate demand for land while its right-hand side represents the unit supply of land.

The model presented in this section makes a number of simplifying assumptions that allow us to focus on the issue of pension funding without imposing further restrictions on preferences, other than those in Assumption 1. Specifically, we take as given expenditures on the public good. We also assume that these expenditures are independent of population size in equation (8) and that old agents do not consume housing. The advantage of these last two assumptions is that the location's current old population is not a state variable of the model. Last, we abstract from explicitly modeling public sector workers as agents in the model. We relax some of these assumptions in section 4.3 in the context of a specific, but convenient, utility function.

2.2 The Demand for Land Under a Binding Downpayment Constraint

Before casting the model in recursive form, it is useful to consider the problem of an agent choosing how much land to purchase as young. In what follows we proceed under the assumption that the downpayment constraint (4) is binding. Notice that the downpayment constraint is always binding if consumption when young is sufficiently more important in utility than consumption when old. For example, this is the case in the special case in which consumption when old is not valued at all, $v'(c_o) = 0$.[15] Replacing the budget constraints

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[15]In Example 1 in Section 3.3 we present an example with a specific utility function and provide sufficient conditions for the downpayment constraint to always be binding. In Appendix B we consider another utility function and provide conditions on the parameters that guarantee that the downpayment constraint is always binding.

(2) and (3) into the objective function (1), the solution to the young agent's optimization problem can be written compactly as:

$$L\left(d_t, q_{t+1}\right) = \underset{l \in [0, w/d_t]}{\arg\max}\, U\left(w - d_t l, l, q_{t+1} l\left(1 - \kappa\right)\right). \tag{11}$$

The function $L\left(d_t, q_{t+1}\right)$ denotes the quantity of land demanded as a function of the down-payment per unit of land d_t and the price of land next period q_{t+1}. When young, the agent acquires $L\left(d_t, q_{t+1}\right)$ units of land at the cost (inclusive of taxes) of $\left(1 + \tau_t\right) q_t$ per unit. Out-of-pocket expenses are only d_t per unit of land because the agent borrows $\kappa q_{t+1}/R$ per unit of land purchased.

The following proposition summarizes the properties of the land demand function.

Proposition 1 *Properties of the demand for land and the indirect utility function.*

(a) *There exists a unique land demand function $L\left(d_t, q_{t+1}\right)$ that solves problem (11).*

(b) *If $u_1\left(c_y, l\right) \rightarrow +\infty$ as $c_y \rightarrow 0$ and $u_2\left(c_y, l\right) \rightarrow +\infty$ as $l \rightarrow 0$, then the demand function $L\left(d_t, q_{t+1}\right)$ satisfies the following first-order condition for l:*

$$-d_t u_1\left(w - d_t l, l\right) + u_2\left(w - d_t l, l\right) + v'\left(q_{t+1}\left(1 - \kappa\right) l\right) q_{t+1}\left(1 - \kappa\right) = 0. \tag{12}$$

(c) *Under the assumptions in part (b), the downpayment constraint binds if and only if the following inequality holds:*

$$u_1\left(w - d_t L\left(d_t, q_{t+1}\right), L\left(d_t, q_{t+1}\right)\right) > v'\left(q_{t+1}\left(1 - \kappa\right) L\left(d_t, q_{t+1}\right)\right) R. \tag{13}$$

(d) *Under the assumptions in part (b), the land demand function $L\left(d_t, q_{t+1}\right)$ is strictly decreasing in d_t. The effect of q_{t+1} on the demand for land is ambiguous.[16] A relaxation of the*

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[16]Specifically, it is strictly decreasing in q_{t+1} if and only if the absolute value of the elasticity of $v'\left(c_o\right)$ with respect to c_o is strictly larger than one.

downpayment constraint (i.e. a higher κ) increases $L(d_t, q_{t+1})$.

(e) Under the assumptions in part (b), the indirect utility function $V(d_t, q_{t+1})$ associated with problem (11) is strictly decreasing in d_t and strictly increasing in q_{t+1}.

A young agent in this economy needs to choose consumption of land as well as consumption of the numeraire when young and when old. Absent the downpayment constraint, the user cost of land should equal the marginal rate of substitution between land and consumption when young and the interest rate would equal the marginal rate of substitution between consumption when young and old (i.e. equation (13) would hold as equality).

In the economy we consider, instead, the agent cannot freely borrow to finance her consumption of land. As a consequence, and differently from the unconstrained case, a marginal increase in the quantity of land demanded results in an increase in consumption when old. This effect is represented by the last term in the first order condition (12). The agent's marginal rate of substitution between consumption when young and old is now larger than the interest rate (equation (13)) as the agent would prefer shifting some old age consumption to the first period of her life. However, doing so would require a reduction in the demand for land (given that borrowing is constrained), which is suboptimal.

The effect of the downpayment on the demand for land is a standard price effect on the demand for a normal good. A higher future price of land produces opposing effects on demand. On the one hand, it makes the investment in land more attractive because it offers a higher return (a substitution effect). On the other, a higher future price of land makes the agent richer increasing the demand for consumption when young (a wealth effect). Given the binding downpayment constraint, the only way for an agent to increase consumption when young is to reduce his demand for land. The latter effect prevails when an increase in old age consumption leads to a relatively large decline in its marginal utility. Interestingly, a relaxation of the downpayment constraint (i.e. a higher κ) always leads to an increase in the demand for land but produces ambiguous effects on consumption when young.

We conclude this section by pointing out that the key difference between the young agent's problem under a binding constraint and the analogous problem when the constraint is not

binding is that in the former the demand for land and indirect utility depend on both d_t and q_{t+1}, while in the latter they depend only on the user cost of land. Thus, absent a downpayment constraint, the equilibrium user cost in a locality, denoted by d_t^*, is uniquely determined by the land market equilibrium (10) condition:

$$P(V(d_t^*))L(d_t^*) = 1. \tag{14}$$

The aggregate demand for land is strictly decreasing in the user cost d_t. Notice that since the equilibrium user cost is independent of the municipal pension funding policy, it must be the case that both young agents' utility and the equilibrium price of land is independent of the local government policy (see Proposition 5 below).

2.3 Recursive Formulation and Definition of Politico-Economic Equilibrium

In this section we cast the model in recursive form and then define a recursive equilibrium without commitment, following Krusell, Quadrini and Rios-Rull (1997), Krusell and Rios-Rull (2000), and Persson and Tabellini (2002).

The state variable for a municipality is the fraction f of pensions that is funded at the beginning of a period. The latter determines the need for current taxes to pay for the promises made in the previous period. Let $f' = F(f)$ denote the funding policy of a municipal government that begins a period with state f. Let $Q(f;F)$ denote the price of land in a municipality that begins a period with state f and whose government follows the policy rule F. Let $D(f';F)$ denote the equilibrium downpayment per unit of land. Notice that the downpayment depends on f' (and not independently on f) because it satisfies the land market clearing condition (equation 10):

$$P(V(D(f';F),Q(f';F)))L(D(f';F),Q(f';F)) = 1, \tag{15}$$

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with $f' = F(f)$. Since the equilibrium downpayment depends on next period's price of land $Q(f'; F)$ and the latter is a function of f', also $D(f'; F)$ depends on f'. Given $Q(f'; F)$, equation (15) admits at most one solution for $D(f'; F)$ because its left-hand side is decreasing in D. Let $T(f; F)$ denote the current period property tax rate in a municipality that follows the funding rule F. The local government's budget constraint in equation (8) can then be re-written as:

$$T(f; F) Q(f; F) = w^g + \frac{f'b^g}{R} + b^g(1 - f) \tag{16}$$

where $f' = F(f)$. The land pricing function and the downpayment function are related by the definition (6), which can also be written as:

$$D(f'; F) = (1 + T(f; F)) Q(f; F) - \kappa Q(f'; F) / R, \tag{17}$$

where $f' = F(f)$.

In what follows, we first define recursively the *economic equilibrium under a given policy rule* for pension funding. We then consider a one-period deviation from this rule and define an *economic equilibrium after a deviation*. Last, we define an *equilibrium without commitment* by imposing that the one-period deviation preferred by the policymaker coincides with the original policy rule.

Definition 1 *Economic equilibrium under a policy rule F.*
Fix the funding rule $f' = F(f)$. An equilibrium under this policy rule is given by the functions $Q(f; F)$, $T(f; F)$, and $D(f'; F)$ such that:

1. *The market for land clears: equation (15) holds.*

2. *The local government's budget constraint, equation (16), holds.*

3. *The downpayment and land pricing function are related by equation (17).*

In order to endogenize the policy rule F, it is necessary to define an equilibrium after a

one-period deviation from that rule.[17] Let $\widetilde{f}'$ denote the funding level, chosen in the current period, that deviates from the policy rule F. A current period deviation will be associated with different current taxes and land prices. Let taxes and current land prices in state f following a one-period deviation $\widetilde{f}'$ from F be denoted by $\widetilde{T}\left(f, \widetilde{f}'; F\right)$ and $\widetilde{Q}\left(f, \widetilde{f}'; F\right)$, respectively. Notice that the young agent faces prices $\widetilde{Q}$ in the current period, but (correctly) anticipates that the pricing function will revert back to Q in the following period. Hence, consumption when old of an agent who is young at the time of the policy deviation depends on the pricing function Q.

With this notation in hand, we can define an economic equilibrium in the municipality after a one-period deviation $\widetilde{f}'$ from the policy rule F:

Definition 2 *Equilibrium after a one-period deviation $\widetilde{f}'$ from the policy rule F.*
An equilibrium after a one-period deviation $\widetilde{f}'$ is given by the functions $\widetilde{Q}\left(f, \widetilde{f}'; F\right)$, $Q\left(f; F\right)$, $\widetilde{T}\left(f, \widetilde{f}'; F\right)$, $D\left(\widetilde{f}'; F\right)$ such that for all $\widetilde{f}'$ the following conditions hold:

1. *The market for land clears:*

$$P\left(V\left(D\left(\widetilde{f}'; F\right), Q\left(\widetilde{f}'; F\right)\right)\right) L\left(D\left(\widetilde{f}'; F\right), Q\left(\widetilde{f}'; F\right)\right) = 1. \qquad (18)$$

2. *The local government's budget constraint holds:*

$$\widetilde{T}\left(f, \widetilde{f}'; F\right) \widetilde{Q}\left(f, \widetilde{f}'; F\right) = w^g + \frac{\widetilde{f}' b^g}{R} + b^g\left(1 - f\right). \qquad (19)$$

3. *The land pricing functions and the downpayment function are related as follows:*

$$D\left(\widetilde{f}'; F\right) = \left(1 + \widetilde{T}\left(f, \widetilde{f}'; F\right)\right) \widetilde{Q}\left(f, \widetilde{f}'; F\right) - \kappa Q\left(\widetilde{f}'; F\right) / R. \qquad (20)$$

Last, we define an equilibrium without commitment for the municipal economy.

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[17]We focus on one-period deviations because each policymaker only controls pension funding and taxes for the period in which she is in power and takes as given the behavior of future policymakers. Thus, focusing on one-period deviations implies that all future policymakers are expected to adhere to the policy rule F.

Definition 3 *Equilibrium without commitment.*

An equilibrium without commitment for the municipality is given by a policy rule F, and set of functions $Q(f;F)$, $T(f;F)$, $D(f';F)$, $\widetilde{Q}\left(f,\widetilde{f}';F\right)$, $\widetilde{T}\left(f,\widetilde{f}';F\right)$ such that:

1. *The functions $Q(f;F)$, $T(f;F)$, $D(F(f);F)$ constitute an economic equilibrium under F according to Definition 1.*

2. *The functions $\widetilde{Q}\left(f,\widetilde{f}';F\right)$, $Q(f;F)$, $\widetilde{T}\left(f,\widetilde{f}';F\right)$, $D\left(\widetilde{f}';F\right)$ constitute an economic equilibrium after a one-period deviation from F according to Definition 2.*

3. *The policymaker has no incentive to deviate from F in any period and for any state, taking into account the economic equilibrium after a one-period deviation. Thus, if the policymaker belongs to the old generation, the consistency requirement is:*

$$F(f) = \arg\max_{\widetilde{f}'} \widetilde{Q}\left(f,\widetilde{f}';F\right) \tag{21}$$

for all f. Alternatively, if the policymaker belongs to the young generation, the consistency requirement is:

$$F(f) = \arg\max_{\widetilde{f}'} V\left(D\left(\widetilde{f}';F\right), Q\left(\widetilde{f}';F\right)\right) \tag{22}$$

for all f.

3   Characterization of Equilibrium

In this section we characterize the equilibrium of the model analytically. We first show in Section 3.1 that the only feasible equilibrium funding rule is a constant. Then we proceed in three steps, corresponding to the three types of equilibria defined in the previous section. In Section 3.2 we characterize the model's equilibrium given an arbitrary (and constant) funding rule $f^* = F(f)$. In Section 3.3 we consider the equilibrium in a locality after a one-period deviation $\widetilde{f}'$ from F. Last, we impose consistency and solve for the equilibrium

without commitment of the model in Section 3.4. In Section 3.5, we conclude by showing that, absent the downpayment constraint, agents' utility is independent of the location's funding policy.

3.1 Constant Funding Rule

In this section we show that the only feasible equilibrium pension funding rule must be a constant:

$$F(f) = f^* \tag{23}$$

for all f. Consider first the case in which the policymaker is a young agent and seeks to maximize lifetime utility, i.e. solve the problem in equation (22). Since the indirect utility function being maximized depends only on $\widetilde{f}'$ and not on f, the solution to this problem must be independent of f.

The old policymaker seeks to maximize current land prices, i.e. solve the problem in equation (21). Differently from the indirect utility function, the land pricing function depends on f. However, it depends on f in a way that does not interact with $\widetilde{f}'$, so the optimal $\widetilde{f}'$ is independent of f. To verify this, use equation (20) to solve for the land pricing function:

$$\widetilde{Q}\left(f, \widetilde{f}'; F\right) = D\left(\widetilde{f}'; F\right) + \kappa Q\left(\widetilde{f}'; F\right)/R - \widetilde{T}\left(f, \widetilde{f}'; F\right) \widetilde{Q}\left(f, \widetilde{f}'; F\right). \tag{24}$$

Then, take into account the government's budget constraint (19) to replace the last term of equation (24) and obtain the following expression for the land pricing function:

$$\widetilde{Q}\left(f, \widetilde{f}'; F\right) = D\left(\widetilde{f}'; F\right) + \kappa Q\left(\widetilde{f}'; F\right)/R - w^g - \widetilde{f}'b^g/R - b^g\left(1 - f\right). \tag{25}$$

Notice that last period's funding f enters additively into this expression and does not interact with $\widetilde{f}'$. It follows that the optimal $\widetilde{f}'$ is independent of f. The following proposition summarizes these results.

Proposition 2 *Constant funding rule:*

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The only possible politico-economic equilibrium of this economy is one in which the funding rule is a constant $F(f) = f^*$ for all f.

3.2 Equilibrium Given an Arbitrary Constant Funding Rule

We begin by solving for the equilibrium of the economy given an exogenous and constant funding policy $f^* = F(f)$ for all f (see Proposition 2). An equilibrium in this case is comprised of a land pricing function $Q(f; f^*)$ and a constant downpayment $D^* = D(f^*; f^*)$ such that the conditions in Definition 1 are satisfied. Notice that when the funding rule is constant, the future land price $Q^* = Q(f^*; f^*)$ is also a constant. It follows from equations (16) and (17) that the equilibrium land pricing function takes the following form:

$$Q(f; f^*) = D^* + \frac{\kappa}{R}Q^* - w^g - \frac{f^* b^g}{R} - b^g(1 - f). \tag{26}$$

In words, the current price of land equals the downpayment D^* plus the discounted future price of land Q^* minus the current taxes needed to pay public sector wages w^g, to fund retirement plans $f^* b^g / R$, and to pay for not-previously-funded pension promises $b^g(1 - f)$. Notice that the relevant discount factor for future land prices in equation (26) is κ/R because the latter determines their impact on the current demand for land when the downpayment constraint is binding.

The equilibrium future price of land Q^* can be obtained by replacing $f = f^*$ in equation (26) and solving for Q^* :

$$Q^* = \frac{D^*}{1 - \kappa/R} - \frac{1}{1 - \kappa/R}\left[w^g + b^g - f^* b^g\left(1 - \frac{1}{R}\right)\right]. \tag{27}$$

The equilibrium downpayment $D(f^*; f^*)$ must be consistent with land market clearing:

$$P(V(D^*, Q^*))L(D^*, Q^*) = 1, \tag{28}$$

taking into account the relationship between Q^* and D^* implied by (27).

In the following proposition we provide sufficient conditions for the existence and uniqueness of equilibrium.

Proposition 3 *Existence and uniqueness of equilibrium with exogenous f^*:*
If the size of the local government is small enough ($w^g \to 0$ and $b^g \to 0$) and the assumptions in part (b) of Proposition 1 hold, then there exists a unique equilibrium of the economy with exogenous f^*.

The proof of uniqueness relies on the fact that both the demand for land by a young agent - the intensive margin effect - and the number of young agents who choose to locate in the municipality - the extensive margin effect - decline as D^* increases, even taking into account the dependence of Q^* on D^* given by equation (27). Therefore, the aggregate demand for land - the left hand side of equation (28) - is decreasing in D^* giving rise to a unique intersection with the vertical supply curve. The intuition is as follows. For given Q^*, as D^* increases, both the demand for land, $L(D^*, Q^*)$, and the utility of locating in the municipality, $V(D^*, Q^*)$, fall (properties (d) and (e) of Proposition 1). The higher downpayment, however, increases the future land price Q^*, with ambiguous effects on $L(D^*, Q^*)$ and a positive one on $V(D^*, Q^*)$ (property (e) of Proposition 1). The reason why the indirect effect of D^* through Q^* cannot be large enough to offset its direct effect on aggregate demand for land is twofold. First, a young agent is constrained, so it discounts the higher future consumption brought about by a higher Q^* at a rate higher than than the interest rate. Second, the complementarity between land and consumption when young in utility ($u_{12}(c_y, l) \geq 0$, Assumption 1, part iii) dampens the response of an agent's demand for land $L(D^*, Q^*)$ to an increase in Q^* : the only way to consume more land is to reduce consumption when young. Thus, uniqueness is implied by the fact that the aggregate demand for land is monotonically decreasing in D^*.

The proof of existence of an equilibrium in Proposition 3 relies on the fact that when D^* is close to zero the demand for land is very large, while when D^* is arbitrarily large the demand for land must be very low due to the high marginal utility of consumption

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when young. These observations, combined with the fact that equation (28) is a decreasing function of D^*, guarantees the existence of a (unique) solution for D^*.[18]

In order to discuss policymakers' incentives to fund pensions, we need to consider a one-period policy deviation from f^*. The next section discusses the impact of such deviation on equilibrium prices and lifetime utility.

3.3   Effects of a One-Period Deviation from Equilibrium

Starting from the equilibrium of the model under a constant policy f^*, consider a current-period deviation $\widetilde{f}'$ by the locality. Since the equilibrium funding rule is the constant f^*, the current deviation has no impact on future funding. Following a deviation, the equilibrium current price of land is given by equation (25). The current price of land depends on the downpayment $D\left(\widetilde{f}'; f^*\right)$ and on the future price of land $Q\left(\widetilde{f}'; f^*\right)$. The latter is given by equation (26) with state variable $\widetilde{f}'$ instead of f because next period the location will have to finance the unfunded portion $1 - \widetilde{f}'$ of pension promises made this period. It follows that in order to characterize the locality's equilibrium after a one-period deviation we only need to determine the response of the downpayment $\widetilde{D}$ to $\widetilde{f}'$. The downpayment is pinned down by the land market clearing equation (18):

$$P\left(V\left(D\left(\widetilde{f}'; f^*\right), Q\left(\widetilde{f}'; f^*\right)\right)\right) L\left(D\left(\widetilde{f}'; f^*\right), Q\left(\widetilde{f}'; f^*\right)\right) = 1. \qquad (29)$$

The left-hand side of this equation represents the aggregate demand for land after a policy deviation. It is given by the product of the young population attracted to the location and the quantity of land demanded by each young agent. Notice that the left-hand side of equation (29) is strictly decreasing in D because of properties (d) and (e) of Proposition 1. Therefore, it uniquely pins down $D\left(\widetilde{f}'; f^*\right)$ as a function of $\widetilde{f}'$ since $Q\left(\widetilde{f}'; f^*\right)$ is a known function of $\widetilde{f}'$.

Figure 1 represents the land market equilibrium condition (29) in a standard demand/supply

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[18]The requirement that the size of the local government is small enough guarantees that the price Q^* remains non-negative as D^* approaches zero.

diagram with the quantity of land on the x-axis and the downpayment D on the y-axis. Each downward-sloping line corresponds to a given deviation $\widetilde{f}'$ from f^*, with the solid line corresponding to the case $\widetilde{f}' = f^*$.
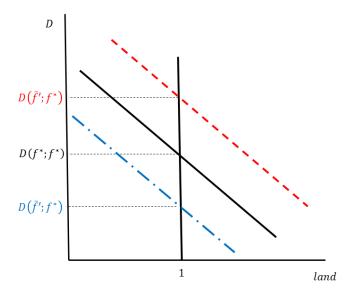

Figure 1: This figure represents the effect of a policy deviation that increases $\widetilde{f}'$ on the equilibrium downpayment. If aggregate demand for land increases with $Q\left(\widetilde{f}'; f^*\right)$ - the case represented by the dashed red line - then the equilibrium downpayment increases, otherwise, if it goes down - the case represented by the dash-dotted blue line - the equilibrium downpayment falls.

Following a deviation that increases $\widetilde{f}'$, the aggregate demand for land - and therefore the equilibrium downpayment - might either increase or decrease. The dashed line in Figure 1 represents the shift in the demand for land following an increase in $\widetilde{f}'$ for the case in which the individual demand for land is increasing in its future price ($L_2(D, Q) > 0$). In this situation the *aggregate* demand for land is also increasing in $Q\left(\widetilde{f}'; f^*\right)$ because the measure of young agents flowing to the location is always increasing in $Q\left(\widetilde{f}'; f^*\right)$.[19] As a result, an

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[19]This stems from the fact that the indirect utility function $V\left(D\left(\widetilde{f}'; f^*\right), Q\left(\widetilde{f}'; f^*\right)\right)$ is increasing in $Q\left(\widetilde{f}'; f^*\right)$ (property (e) of Proposition 1) and the function $P(V)$ is increasing in V.

increase in $\widetilde{f}'$ leads to a higher equilibrium downpayment. The dashed-dotted line in Figure 1 represents the alternative case in which the individual demand for land is decreasing in $Q\left(\widetilde{f}'; f^*\right)$ and this effect is strong enough to make the *aggregate* demand for land decrease in $Q\left(\widetilde{f}'; f^*\right)$ as well. As a result, in this case, an increase in $\widetilde{f}'$ leads to a lower equilibrium downpayment.

Despite this ambiguity, it is possible to show that the equilibrium downpayment cannot increase "too much" in response to a higher $\widetilde{f}'$. The following lemma specifies what this means.

Lemma 1 *Upper bound on downpayment effect*:

The largest possible increase in the equilibrium downpayment following an increase in $\widetilde{f}'$ is given by:

$$\frac{\partial D}{\partial \widetilde{f}'} \leq \frac{v'\left(Q\left(1-\kappa\right)L\right)}{u_1\left(w - DL, L\right)}\left(1-\kappa\right)b^g, \tag{30}$$

where $Q = Q\left(\widetilde{f}'; f^*\right)$, $D = D\left(\widetilde{f}'; f^*\right)$ and $L = L\left(D, Q\right)$. This inequality is strict if and only if $P'\left(V\right) < +\infty$ and at least one of the following strict inequalities hold: $u_{11}\left(c_y, l\right) < 0$, $u_{12}\left(c_y, l\right) > 0$, $v''\left(c_o\right) < 0$.

The expression on the right-hand side of equation (30) represents an agent's willingness to pay for a marginal increase in $\widetilde{f}'$. The latter reduces future taxes by b^g, increases future land prices by the same amount, and consumption when old by $\left(1-\kappa\right)b^g$. Notice that the willingness to pay reflects the agent's marginal rate of substitution between consumption when young and consumption when old. This is an *upper* bound for the increase in the equilibrium downpayment, instead of being exactly equal to it, because there are two margins of response to a higher future price of land induced by $\widetilde{f}'$. First, the utility offered by the location increases, leading to an inflow of young agents. Second, the demand for land per young agent varies in response to higher future land prices.[20] Following these responses, the downpayment adjusts to restore land market equilibrium. If land demand per young agent

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[20]Recall that the demand for land might either increase or decrease in response to an increase in the future price of land.

was constant, the downpayment would have to increase exactly by the amount on the right-hand side of equation (30) in order to perfectly offset the increased inflow of population. On the other hand, if population was constant, the downpayment would have to change by the amount necessary in order to keep the demand of land per capita constant in response to a higher $\widetilde{f}'$. The latter effect is always smaller than the former by the concavity of the utility function and the complementarity between land and consumption when young, $u_{12}(c_y, l) \geq 0.$[21] This explains why the expression on the right-hand side of equation (30) is an upper bound.

The result in Lemma 1 allows us to determine the effect of a policy deviation $\widetilde{f}'$ on the equilibrium price of land and on the lifetime utility of a young agent.

Effect of policy deviation on the current price of land. The land pricing function following a deviation is given by equation (25). In order to evaluate the effect of $\widetilde{f}'$ on $\widetilde{Q}\left(f, \widetilde{f}'; f^*\right)$, take the partial derivative of equation (25) with respect to $\widetilde{f}'$:

$$\frac{\partial \widetilde{Q}\left(f, \widetilde{f}'; f^*\right)}{\partial \widetilde{f}'} = - \underbrace{\frac{b^g}{R}}_{\text{current taxes}} + \underbrace{\frac{\kappa b^g}{R}}_{\text{borrowing}} + \underbrace{\frac{\partial D\left(\widetilde{f}'; f^*\right)}{\partial \widetilde{f}'}}_{\text{downpayment}}. \tag{31}$$

The net effect of a policy deviation $\widetilde{f}'$ on the current price of land depends on the three terms on the right-hand side of equation (31). The first term represents the effect of the higher current taxes associated with an increase in $\widetilde{f}'$ on the price of land. A marginal increase in $\widetilde{f}'$ causes current property taxes to increase by b^g/R. The latter are capitalized in (lower) contemporaneous land prices on a one-for-one basis. The second and third terms on the right-hand side of equation (31) represent the effects of lower taxes next period - induced by a higher $\widetilde{f}'$ - on current land prices. Specifically, the second term captures the fact that a young agent can borrow $\kappa b^g/R$ units of consumption as a response to a reduction in future taxes by b^g, because the price of land when old increases by b^g as well. The third term

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[21]This is obvious when the demand for land decreases in response to a higher future price (the partial derivative $L_2(D, Q) \leq 0$). In this case, the equilibrium downpayment needs to fall to re-establish land market equilibrium.

on the right-hand side of equation (31) reflects the fact that, even if young agents cannot borrow against the portion $(1 - \kappa)$ of the increase in future land prices, the latter nevertheless affects the attractiveness of the location and the incentives to purchase land there. In other words, the increase in future land prices induced by $\widetilde{f}'$ affects the equilibrium downpayment proportionately to the fraction $(1 - \kappa)$ of land's value that cannot be collateralized.

The net of these three effects becomes clear after replacing the upper bound for the change in the downpayment from Lemma 1 into equation (31):

$$\frac{\partial \widetilde{Q}\left(f, \widetilde{f}'; f^*\right)}{\partial \widetilde{f}'} \leq -\frac{b^g}{R} + b^g \left[\frac{\kappa}{R} + \frac{1 - \kappa}{u_1\left(w - DL, L\right)/v'\left(Q\left(1 - \kappa\right)L\right)}\right] < 0.$$

The combined effect of the second and third terms on the right-hand side of equation (31) cannot exceed the direct effect of the first one because young agents discount the non-collateralizable portion $(1 - \kappa)$ of the increase in future land prices using their marginal rate of substitution for consumption rather than the interest rate R. The former is higher than the latter because of the binding downpayment constraint (part (c) of Proposition 1). It follows that the equilibrium land price falls in response to an increase in pension funding $\widetilde{f}'$.

Effect of policy deviation on the lifetime utility of a young agent. As we have shown above, an increase in current taxes compensated by a reduction in future taxes has a limited effect on the equilibrium downpayment (Lemma 1), while increasing land prices in the future. As a result, an increase in pension funding benefits young agents. More formally, a young agent's indirect utility $V\left(D\left(\widetilde{f}'; f^*\right), Q\left(\widetilde{f}'; f^*\right)\right)$ would, by definition, remain constant after an increase in $\widetilde{f}'$ only if the equilibrium downpayment increased exactly by the non-collateralizable portion $(1 - \kappa)$ of the increase in future land prices discounted at her consumption marginal rate of substitution. Since this increase in the downpayment coincides with Lemma 1's upper bound, a young agent must be (weakly) better off following an increase in pension funding $\widetilde{f}'$.

The following proposition summarizes our findings regarding the effect of a policy devia-

tion in land prices and lifetime utility.

Proposition 4 *Impact of a policy deviation*:
A current-period deviation $\widetilde{f'}$ that increases funding above f^* leads to strictly lower current land prices and a weakly higher lifetime utility of locating in the municipality:

$$\frac{\partial \widetilde{Q}\left(f, \widetilde{f'}; f^*\right)}{\partial \widetilde{f'}} < 0 \tag{32}$$

$$\frac{\partial V\left(D\left(\widetilde{f'}; f^*\right), Q\left(\widetilde{f'}; f^*\right)\right)}{\partial \widetilde{f'}} \geq 0. \tag{33}$$

The second inequality is strict if the sufficient condition for a strict inequality in Lemma 1 is satisfied.

What role does geographic mobility of young agents play in giving rise to the results of Proposition 4? Intuitively, geographic mobility should act as a force that dampens the effect of pension underfunding on young agents' utility and on the price of land. This intuition is correct within the context of our model, although geographic mobility does not prevent pension underfunding from increasing local land prices. In the extreme case in which labor mobility is perfect - in the sense that a locality would not be able to attract *any* young agent if it offered less than some lifetime utility $\overline{V}$ - young agents' utility is insulated from any attempt to underfund pensions. In such case, which is not covered by Proposition 4 because it violates Lemma 1's assumption that $P'(V) < +\infty$, the equilibrium downpayment must unambiguously decline by the amount on the right-hand side of equation (30) in order to fully compensate young agents for the decline in future land prices induced by $\widetilde{f'}$. Even in this case, however, the net effect on the locality's price of land is positive. This is because, as explained above, the decline in the downpayment reflects a constrained agent's willingness to pay today for a marginal increase in consumption when old, while the shifting of taxes to the future allows the locality to reduce current taxes by $1/R$ dollars for each dollar of future

taxes. Since a young agent is constrained, the former effect is dominated by the latter even with perfect mobility of labor.

Example 1 In this section we introduce an example to illustrate some of the points made above. Consider the following utility function:

$$U = c_{yt} + \phi\left(l_t\right) + \beta c_{ot+1}, \tag{34}$$

with $\phi'\left(l\right) > 0$ and $\phi''\left(l\right) < 0$. Assume that $\beta R < 1$, so that the agent is always constrained. It is straightforward to derive the demand for land and the indirect utility function as a function of the current downpayment $D = D\left(\widetilde{f}'; f^*\right)$ and the future price of land $Q = Q\left(\widetilde{f}'; f^*\right)$:

$$
\begin{aligned}
L\left(D, Q\right) &= \phi'^{-1}\left(D - \beta\left(1 - \kappa\right)Q\right), \tag{35}\\
V\left(D, Q\right) &= w - \phi'\left(L\left(D, Q\right)\right)L\left(D, Q\right) + \phi\left(L\left(D, Q\right)\right). \tag{36}
\end{aligned}
$$

Imposing the land market equilibrium equation (10) then pins down the argument of the land demand function:

$$D\left(\widetilde{f}'; f^*\right) - \beta\left(1 - \kappa\right)Q\left(\widetilde{f}'; f^*\right) = z^*, \tag{37}$$

where z^* is a policy-independent constant such that:

$$P\left(w - z^*\phi'^{-1}\left(z^*\right) + \phi\left(\phi'^{-1}\left(z^*\right)\right)\right)\phi'^{-1}\left(z^*\right) = 1. \tag{38}$$

The condition (37) allows us to easily compute

$$\frac{\partial D\left(\widetilde{f}'; f^*\right)}{\partial\widetilde{f}'} = \beta\left(1 - \kappa\right)b^g. \tag{39}$$

Notice that this corresponds to the term on the right-hand side of equation (30) because for the utility function (34) the marginal rate of substitution of consumption across periods is

simply β. Notice also that in this case the upper bound in equation (30) is attained because the necessary conditions for a strict inequality in Lemma 1 are not satisfied. Replacing equation (39) into the expression for the current land price in equation (31) yields:

$$\frac{\partial \widetilde{Q}\left(f, \widetilde{f}'; f^*\right)}{\partial \widetilde{f}'} = (1 - \kappa) b^g \left(\beta - \frac{1}{R}\right) < 0 \tag{40}$$

because $\beta R < 1$. Thus, an increase in funding lowers current land prices because the increase in the equilibrium downpayment is smaller than the net effect of increasing current taxes while reducing them in the future. By contrast, there is no effect of $\widetilde{f}'$ on young agents' utility because the latter depends only on the term on the left-hand side of equation (37), which is a constant. Notice that the sufficient condition in Proposition 4 for a strictly positive effect of $\widetilde{f}'$ on young agents' utility are not satisfied. What is going on is that in response to a higher funding level $\widetilde{f}'$, the equilibrium downpayment offsets exactly any utility gain associated with higher future land prices.

3.4 Intergenerational Conflict Over Funding Decisions

In this section we impose the consistency conditions (21) and (22), and solve for the equilibrium funding policy under two alternative assumptions about the identity of the policymaker. Specifically, we consider the case in which the policymaker is an old agent and then the case in which it is a young one. Considering these two extreme cases serves to highlight the intergenerational tension over pension funding.

When currently old agents set the funding policy, the objective is simply the maximization of the price of land. The higher the price at which an old agent is able to sell her land to the incoming young agents, the higher her current consumption is. A young policymaker, instead, maximizes her lifetime utility.[22]

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[22]Notice that the reason why the equilibrium solution is always a corner is that political power is assumed to be concentrated in the hand of one group. More generally, we could assume that the policymaker's preferences are a weighted average of the utility of the young and old groups. In this case, however, we would need to include an additional state variable to keep track of the size of the young population in the economy.

Corollary 1 then directly follows from Proposition 4:

Corollary 1 *Equilibrium policies:*

(1) If an old agent sets the funding policy, the only politico-economic equilibrium is one in which pension funding is the minimum allowed, or $f^* = f_{\min}$.

(2) If a young agent sets the funding policy and if the condition for a strict inequality in Lemma 1 is satisfied, then the only politico-economic equilibrium is one in which pensions are fully funded, or $f^* = 1$.

How would two otherwise identical localities controlled by young and old agents look like? Given an arbitrary initial state f inherited from the past, a locality subsequently controlled by old agents would be characterized by *higher* land prices $Q(f; f_{\min})$ in that period, and *lower* land prices in subsequent periods $Q(f_{\min}; f_{\min})$ relative to a locality controlled by young agents. The locality controlled by old agents would be characterized by a lower lifetime utility for young agents $P(V(D(f_{\min}; f_{\min}), Q(f_{\min}; f_{\min})))$, and therefore a smaller young population in all periods. By contrast, absent the downpayment constraint, the funding policy would have no effect on current land prices, in the sense that $Q(f; f_{\min}) = Q(f; 1)$. It would also have no effect on the size of the location's young population. However, also in this case, underfunding would be reflected in lower land prices in subsequent periods, or $Q(f_{\min}; f_{\min}) < Q(1; 1)$ because starting a period with more unfunded liabilities inherited from the past necessitates higher property taxes today.

3.5 No Downpayment Constraint

If the downpayment constraint does not bind (or if it is not present), a policymaker cannot affect either the utility of the old generation or the utility of the young generation by un-derfunding the pension system. In this case, as already mentioned at the end of Section 2.2, the demand for land and consumption when young and old depend only on the *user cost of land*, which is defined as d in equation (6) with the parameter κ set equal to 1. Since the future price of land Q does not play an independent role in affecting agents' choices (given

the user cost of land), a policy deviation $\widetilde{f}'$ has no effect on the equilibrium user cost of land, consumption, or population. In such situation, the derivative $\partial D/\partial \widetilde{f}'$ is equal to zero by definition. Moreover, with a perfect capital market the increase in current taxes is perfectly compensated by the decline in the future price of land induced by higher future taxes. This point can be verified by replacing $\kappa = 1$ in the second term on the right hand side of equation (31) and cancelling it out with the first one. The following proposition summarizes this result.

Proposition 5 *Policy deviation if the downpayment constraint does not bind:*
Without a downpayment constraint (or when the latter does not bind), both the price of land and the indirect utility offered by a municipality are independent of the one-period deviation $\widetilde{f}'$ from f^*. As a result, both young and old agents are indifferent about alternative pension funding policies.

4 Discussion and Extensions

In this section we discuss a number of variants of the basic model in order to highlight the robustness of our results and some of the key assumptions behind them.

4.1 No Downpayment Constraint but Higher Borrowing Rate

A key aspect of our analysis so far is the fact that changes in the future price of land are reflected in the current downpayment by a factor κ/R (see equation (6). Since $\kappa < 1$, this implies that, for given downpayment, a reduction in current taxes by \$1 offset by a corresponding increase in future taxes by \$$R$ leads to an increase in current land prices by \$$(1 - \kappa)$. It might be argued - correctly - that a similar result would obtain if young agents were not constrained, but faced a borrowing rate R^b larger than the rate R at which the government saves. In this case both demand for land and indirect utility would only depend on the user cost of land - given by equation (17) with R^b replacing R - and not independently on its future price. The land market equilibrium condition would then pin down the user

cost. As equation (14) makes clear the equilibrium user cost is independent of any policy variable. Therefore, the utility level that a city would afford to a young agent would also be independent of the locality's funding policy. Thus, differently from our Proposition 4 result, young agents would be indifferent among alternative funding policies. As anticipated above, old agents, would rather not fund the pension system in order to increase current land prices. The latter would be given by the following version of equation (31):

$$\frac{\partial \widetilde{Q}\left(f, \widetilde{f}'; f^*\right)}{\partial \widetilde{f}'} = -\frac{b^g}{R} + \frac{b^g}{R^b}.$$

The sign of this expression is negative because $R^b > R$. To summarize, in this version of the model there is no tension between generations: the old prefer to underfund, while the young are indifferent. Therefore, a key implication of the downpayment constraint is to make the young averse -rather the indifferent - to pension underfunding.

4.2   Role Played by the Price Elasticity of Housing Supply

Suppose we had a perfectly elastic supply of land in the location at the price:

$$Q = \overline{Q}.$$

It follows that old agents cannot affect $\overline{Q}$ so they are indifferent about pension funding. The downpayment is given by equation (25):

$$D\left(\widetilde{f}'; f^*\right) = \overline{Q} + w^g + \frac{\widetilde{f}' b^g}{R} + b^g \left(1 - f\right) - \frac{\kappa}{R}\overline{Q}.$$

Notice that the downpayment increases with higher levels of pension funding because of the higher taxes. It follows that a young agent's utility

$$V\left(D\left(\widetilde{f}'; f^*\right), \overline{Q}\right)$$

31

is decreasing in funding levels. In summary, without capitalization effects, if the old agents are in control they are indifferent among various funding policies. If the young ones are in control, instead, they prefer not to fund the pension system. This result is independent of the downpayment constraint. This implies that as we increase the share of old households we should get a smaller degree of underfunding in more elastic cities.

4.3   Municipal Workers and Endogenous Public Good

The model we introduced in Section 2 abstracts from explicitly considering public sector workers and from endogenizing the quantity of the public good the local economy consumes. In this section we show how to extend our economy in this direction for the special case in which agents' utility takes the form

$$U = c_{yt} + \phi\left(l_t\right) + \lambda\left(g_t\right) + \beta c_{ot+1}, \tag{41}$$

where young agents derive utility $\lambda\left(g_t\right)$ from consumption of the public good g. The function $\lambda\left(g\right)$ is such that $\lambda'\left(g\right) > 0$ and $\lambda''\left(g\right) < 0$. Notice that this is a generalization of the utility function in Example 1. The public good is produced by public sector workers according to the production function:

$$g_t = n_t^g,$$

where n_t^g denotes the measure of public sector workers. The latter have the same utility function (41) as regular agents. They are also subject to the downpayment constraint (4) on housing. Public sector workers are paid a package that includes an exogenous wage w^g and a promised pension benefit b^g that accrues when they are old. We assume that (w^g, b^g) is such that:

$$w = w^g + \beta b^g,$$

so a young agent is indifferent between working in the private sector at the wage w or in the government sector. Thus, public sector workers solve the same problem as regular agents.

The government's budget constraint is now given by:

$$\tau_t q_t = g_t \left(w^g + \frac{f_{t+1} b^g}{R} \right) + g_{t-1} b^g \left(1 - f_t \right).$$

The indirect utility function is now:

$$V \left(z_t, g_t \right) = \max_l \left[w - z_t l + \phi \left(l \right) + \lambda \left(g_t \right) \right],$$

where z_t is defined as $z_t \equiv d_t - \beta \left(1 - \kappa \right) q_{t+1}$. Land market clearing requires:

$$P \left(V \left(z_t, g_t \right) \right) L \left(z_t \right) = 1.$$

This equation pins down $z_t = z \left(g_t \right)$, with $z' \left(g_t \right) > 0$ and $z'' \left(g_t \right) < 0$ because more public good increases utility and aggregate demand for land at a decreasing rate.

Following the same steps as in Section 3, it can be shown that the equilibrium policy is a constant (f^*, g^*) such that if old agents are in charge of policy, $f^* = 0$ and g^* such that:

$$z'(g^*) = w + \kappa \left(\frac{1}{R} - \beta \right).$$

As before, a young agent is indifferent with regard to the funding policy. The optimal choice of g^* is implicitly defined by:

$$z'(g^*) L \left(z \left(g^* \right) \right) = \lambda' \left(g^* \right).$$

4.4 Property Taxes vs Income Taxes

Our results are sensitive to the tax instrument available to the local government. Suppose that instead of property taxation the local government can use a lump-sum tax denoted by T_t. Also, to simplify the analysis, suppose that utility is given by equation (34) and that the locality's population is fixed and of measure one. Given the form of the utility function, the

demand for land takes the same form as in equation (35) and the land market equilibrium condition $L(z) = 1$ pins down z^* as defined in equation (37). As a consequence, the price of land is the constant:

$$Q^* = \frac{z^*}{1 - \frac{\kappa}{R} - \beta(1 - \kappa)}.$$

Old agents are therefore indifferent about pension funding. Young agents care about their indirect utility given by:

$$V(T) = w - T - \phi'(1) + \phi(1),$$

where T is given by the government's budget constraint:

$$T = w^g + \frac{f' b^g}{R} + b^g(1 - f).$$

Thus, young agents prefer to set f' to its minimum possible value $f_{\min}$. Notice that this is the opposite result with respect to the one in Example 1. With the income tax, the young prefer to underfund and the old are indifferent. The difference between the two cases has to do with the fact that the income tax is not capitalized in land prices because the demand for land is independent of the income tax. As a consequence, the equilibrium land price is independent of the income tax. Lack of capitalization implies that the young are left to pay the income tax required to fund promised pensions as well as the pensions promised in the previous period. Given that they do not perceive any benefit from funding pensions, the young optimally choose to leave them unfunded.[23]

5   Some Policy Implications

The discussion above has highlighted a tension between the interests of the old and of the young generations with regard to funding pensions. Old agents prefer to underfund the system as much as possible, while young agents prefer to fully fund it. In this section we

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[23]Some capitalization of income taxes would obtain in a version of this example with endogenous population. A higher income tax would reduce utility for prospective residents and thus reduce a location's population and the demand for land.

consider this conflict from the perspective of a higher level of government that is in charge of setting the minimum level of pension funding, $f_{\min}$. In the U.S., state governments have adopted different laws and regulations about local pension funding. For example, in 1994 the Federal government's Governmental Accounting Standards Board introduced the concept of Annual Required Contribution (ARC) to provide an estimate of the flow contributions needed to "adequately" fund a defined benefits pension plan. Brainard and Brown (2015) point out that pension funds are not required by law to contribute the ARC and write that "laws and practices governing payment of pension contributions vary widely among states."[24] They show that states that have encoded the ARC in their statutes and laws have in recent years made larger contributions to their defined benefits pension plans than states where pension contributions are left at the discretion of plan administrators and policymakers. Consistent with this narrative, we interpret the $f_{\min}$ parameter as a federal or state mandate to minimally fund their pension promises.

Since young agents choose to fully fund the system, in this circumstance the minimum funding policy has no impact on equilibrium utilities. When old agents set the policy, instead, $f_{\min}$ matters. The following proposition summarizes our results.

Proposition 6 *State or federal mandate regarding minimum funding level.*
Assume that the condition for a strict inequality in Lemma 1 is satisfied. Then:
(a) If old agents set the funding policy, the utility of the first (old) generation is decreasing in $f_{\min}$, while the lifetime utility of all other generations is strictly increasing in $f_{\min}$.
(b) If young agents set the funding policy, $f_{\min}$ has no effect on any agent's utility.

When the old set the policy, a higher $f_{\min}$ leads to a higher level of f^*. In turn, a higher f^* lowers the *current* price of land $Q\left(f; f^*\right)$, and increases the *future* price $Q\left(f^*; f^*\right)$. The former effect is damaging to the initial old generation. The latter favors subsequent generations, starting from the initial young. Notice that, while in response to a higher future

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[24]Specifically, they write that "some states require that the amount recommended by the retirement system actuary be paid; some states consistently pay the amount recommended by the retirement system actuary, even if it is not legally required; other states appropriate pension contributions in amounts that are not linked to an actuarial calculation. Still other states base their contributions on a statutorily fixed rate, such as a percentage of employee payroll."

land price $Q\left(f^*; f^*\right)$, the equilibrium downpayment $D\left(f^*; f^*\right)$ might increase, this effect and its impact on lifetime utility is dominated by the direct effect associated with a higher $Q\left(f^*; f^*\right)$. The intuition is analogous to the one associated with Proposition 4.

If the assumption of Proposition 6 does not hold, a minimum funding policy can make the initial old worse off without increasing the utility of the subsequent generations. This is the case in example 1.

Example 1 (continued) *Minimum funding in Example 1. The equilibrium current price of land when the old generation sets the funding policy is given by:*

$$Q\left(f; f_{\min}\right) = \frac{z^* - w^g - b^g}{1 - \kappa/R - \beta\left(1 - \kappa\right)} - f_{\min}\frac{b^g}{1 + \frac{R-1}{(1-R\beta)(1-\kappa)}} + fb^g.$$

This is decreasing in $f_{\min}$ because $\beta R < 1$. It follows that requiring the locality to increase the minimum funding $f_{\min}$ leads to lower current land prices, hurting the initial old, and without affecting any other cohort.

6   Evidence on the Relationship between Underfunding and Capitalization

This section presents some empirical support for the key conclusions of our theoretical analysis. The key assumption guiding our empirical analysis is that the preferences of young agents are more likely to be reflected in a location's funding policy in municipalities characterized by a larger share of young households. We provide evidence that municipalities with younger populations, on average, have lower levels of unfunded liabilities (Corollary 1). Furthermore, this relationship strengthens in cities that are relatively more land constrained. In the most sparsely populated cities, on the other hand, which presumably have more ability to add housing, a younger population is associated with higher unfunded liabilities (Section 4.2).

In order to conduct the analysis, we draw data from several sources. The first is a report prepared by Munnell and Aubry (2016), that calculates unfunded actuarial accrued liabilities

(UAAL) for 173 large cities.[25] This report also provides data on the ratio of UAAL to own-source revenues. In addition, we collect data from the U.S. Census to calculate measures of age distribution, home ownership, home values, income, population density, and population growth.[26]

We normalize UAAL by different economic variables to create several measures of pension liabilities: UAAL per capita, UAAL divided by aggregate income, UAAL divided by own-source revenues, and UAAL divided by aggregate housing value. For a measure of the age distribution we use the percentage of total households that are headed by owners under 55 years old.[27] Finally, we use residential population density as an inverse proxy for land supply elasticity.[28]

Table 1 provides summary statistics for the data. Unfunded liabilities vary greatly among cities. The average unfunded liabilities per capita are $1,446$, and range from a high of $8,775$ in New York, to a low of negative 463 in Seattle, indicating a surplus. The pattern is similar when liabilities are measured against aggregate income, own-source revenues, and aggregate house values.[29] On average, 28.4 percent of households in cities are headed by homeowners under the age of 55. Finally, density ranges from a high of 27,092 people per square mile in New York to a low of 171 Anchorage. The natural log of density is used in

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[25]The authors of the report collected data from the 2012 Comprehensive Annual Financial Reports (CAFR) of 173 selected U.S. cities. The cities were selected to create a sample that included large cities in each state and provide some variation in institutional arrangements. The authors calculated UAAL using new Governmental Accounting Standards Board (GASB) guidelines implemented in 2015, which require more transparent reporting of pension liabilities by local municipalities.

[26]Data was drawn from the 2012 American Community Survey and the 1980 Decennial Census. Because of differences in municipal definitions, a few cities were dropped when merging the data, leaving a sample of 168 cities.

[27]Notice that this specification does not distinguish between renters and home-owners over 55. It might be argued that renters might have different preferences for pension funding relative to homeowners. We have also considered versions of our main regression that distinguish renters from homeowners. The results are qualitatively similar to those in Table 2.

[28]For this investigation, we would not want to use measures of metropolitan area supply elasticities, such as those estimated by Saiz (2010) and others, given that we are studying municipalities instead of metropolitan areas. Land and housing constraints faced by municipalities are largely driven by how their borders are drawn, whether they can annex surrounding property, and whether or not they are relatively more urban or suburban in form. Population density is presumably a good proxy for this. Formally, we are assuming that the marginal costs of supplying additional housing are increasing in population for a given land area.

[29]The rank correlation coefficient across our four measures of unfunded liabilities is always above 0.80.

subsequent regressions, so those statistics are also provided.

Table 1: Summary of data

	Mean	Std. Dev.	Min	Max
UAAL/population ($)	1,446	1,416	-463	8,775
UAAL/annual income (%)	6.21	6.11	-1.22	29.74
UAAL/annual revenue (%)	71.6	66.3	-17.0	359.0
UAAL/house value (%)	3.81	4.18	-0.71	22.63
Owners under 55 (%)	28.4	6.5	12.0	52.5
Population density (per sq. mi.)	3,993	3,481	171	27,092
ln density	8.03	0.72	5.14	10.21

Source: Data on unfunded actuarial accrued liabilities (UAAL) and revenues are from Munnell and Aubry (2016). Data on population, income, house values, age, ownership, and density are drawn from the U.S. Census, 2012 Annual Community Survey.

The first notable relationship that arises in the data is that the age distribution of the population in different cities is correlated with the level of pension liabilities. Figure 2 plots the homeowners under the age of 55 as a percentage of total households versus UAAL per capita for each city in the sample. There is a strong negative correlation between the percentage of young homeowners in a city and the level of unfunded liabilities. In order to further investigate this correlation we turn to regression analysis.

Table 2 shows the results of regressions using the four measures of pension liabilities as dependent variables with various specifications. The first column in each panel shows the results including only the percentage of young owners. The results show that as the percentage of young homeowners increases, the level of the unfunded liabilities declines. The economic interpretation in terms of UAAL per capita is that a one percentage point increase
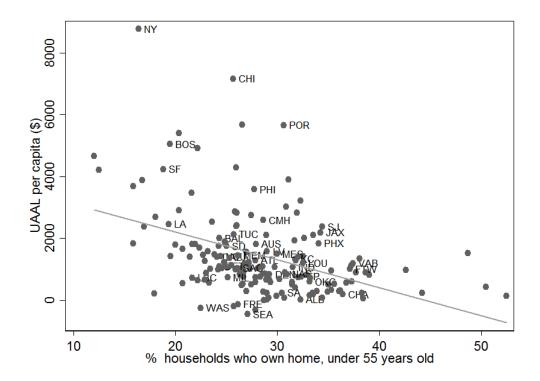
Figure 2: This figure plots the percentage of households who own their home and are headed by someone under 55 years old versus UAAL per capita for 168 large U.S. cities.

in the share of young homeowners decreases unfunded liabilities by \$90.6 per capita. The negative correlation between share of young homeowners and unfunded liabilities could be driven by the fact that economically declining cities are populated by older households and have larger unfunded liabilities. In order to partially control for these effects, in the second column of each panel we control for Census region dummies, a city's population growth between 1980 and 2012, and a number of other variables, including population density.[30] The effect of age distribution is diminished but still significant. In the third column, we include an interaction term between the age variable and density. For three out of four measures of pension liabilities, the coefficients on the interaction terms are negative and significant. This shows that the relationship between age distribution and pension funding is more pronounced in locations where housing supply is more inelastic. More precisely, in

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[30] Other controls include the ratio of median income and median house values, and the natural log of population.

denser cities, a larger proportion of young homeowners is associated with decreased liabilities, whereas, in the most sparse cities, more young owners are associated with increased liabilities. Finally, in the fourth column, we use the same specification, but again include a number of controls. These additional controls have little effect on the estimates. Taken together, these results are consistent with the predictions of our model.

7 Conclusions

In this paper we have explored the political and economic determinants of underfunding of municipal pension plans using a new dynamic politico-economic model. The key insight of our the model is that pension funding policies produce distributional effects across generations if agents are subject to binding downpayment constraints when purchasing land. In such situation, young and old policymakers disagree on the funding policy to pursue, with the former favoring full funding and the latter favoring underfunding. As a result, state-wide policies that mandate binding minimum funding levels hurt the initial old and benefits subsequent cohorts. Empirical results based on cross-city comparisons in the magnitude of unfunded liabilities are consistent with the key prediction of the model.

According to our results, realistic housing market frictions imply that local debt matters for agents' utility. We hypothesize that this result extends beyond the specific question of pension funding analyzed here. For example, it has been shown by Conley and Rangel (2001) in the context of an overlapping generations model with perfect capital markets, that land price capitalization provides the correct incentives for old generations to invest efficiently in durable public goods and that agents' utility is unaffected by whether this investment is financed through debt or taxes. We have shown that the latter result does not hold in our model. We hypothesize that the former result may not hold either for the same reason why debt financing leads to higher current land prices in our model: while the benefits of a higher quantity of durable public goods available to future generations may be capitalized into future land prices, a binding downpayment constraint implies that higher future land

Table 2: Determinants of Underfunding of Municipal Pensions

	UAAL/population				UAAL/income			
% owners U55	-90.6**	-41.4**	493.7**	346.1**	-0.43**	-0.27**	1.72**	1.41**
	(17.5)	(13.9)	(181.1)	(168.2)	(0.08)	(0.07)	(0.68)	(0.71)
ln(density)		224.8**	2423.5**	1624.3**		1.26*	8.99**	7.32**
		(156.2)	(705.8)	(674.5)		(0.70)	(2.60)	(2.95)
% owners U55 $*$ ln(density)			-68.8**	-48.7**			-0.25**	-0.21**
			(23.0)	(21.7)			(0.09)	(0.09)
controls^a		y		y		y		y
R^2	0.17	0.31	0.30	0.33	0.21	0.34	0.31	0.36
observations	168	165	168	165	168	165	168	165
	UAAL/revenues				UAAL/house values			
% owners U55	-2.55**	-1.64*	0.44	-2.72	-0.29**	-0.25**	0.82	0.92*
	(0.75)	(0.86)	(7.13)	(8.14)	(0.06)	(0.06)	(0.50)	(0.52)
ln(density)		16.03**	30.32	12.14		0.80	4.41**	5.05**
		(6.88)	(25.79)	(32.02)		(0.49)	(1.90)	(2.14)
% owners U55 $*$ ln(density)			-0.23	0.14			-0.13**	-0.15**
			(0.92)	(1.06)			(0.07)	(0.07)
controls		y		y		y		y
R^2	0.06	0.16	0.12	0.16	0.20	0.36	0.24	0.39
observations	168	165	168	165	168	165	168	165

*—p<0.1, **—p<0.05

^{a}Controls include the percentage population change between 1980 and 2012, dummies for the four Census regions, the ratio of median income and median house values, and the natural log of population.

prices lead to higher current land prices only to the extent that young agents are able to borrow against the former. We leave this and other interesting extensions of our model for future research.

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A   Pension Funding Across U.S. Cities

In this section we briefly document the magnitude of unfunded pension liabilities in large U.S. cities. Table A.1 shows liabilities and funding levels of defined benefit plans of public employees for the 20 largest U.S. cities in 2009. The unfunded portion of pension liabilities for these twenty cities alone totals $85.5 billion. The level of pension funding varies across cities, from Chicago's pension liabilities, which are only 52 percent funded, to San Francisco, at 97 percent. New York has the highest unfunded liabilities per capita at $5,453, and unfunded liabilities are significant when compared to annual general revenues, with Chicago topping the list with a ratio of 1.69. While comprehensive funding data is not readily available for all municipalities in the U.S., this phenomenon is not confined to large urban central cities. For example, according to the 2012 Status Report on Local Government Pensions released by the Public Employee Retirement Commission of Pennsylvania, 630 out of 3,161 local pension plans in Pennsylvania were less than 80 percent funded. On net, the $28 billion in accrued liabilities for local Pennsylvania governments were only 70 percent funded.

Further exacerbating the issue is the fact that most reported liabilities are best-case scenarios, given that optimistically high discount rates are often used to calculate their present value. Novy-Marx and Rauh (2011) found that reported state pension liabilities might be underestimated by up to 30 percent. Recently, the Government Accounting Standards Board, which sets accounting standards for local governments, set new guidelines for calculating and reporting pension liabilities which will most likely lead to changes in reported funding levels, worsening the outlook for municipalities.

B   Utility Function and Binding Downpayment Constraint

In this section we consider a special utility function for which it is feasible to derive conditions on parameters such that the downpayment constraint is always binding. We first focus on

Table A.1: Pension Liabilities and Funding Levels for 20 largest U.S. Cities, 2009

	Total Liabilities ($ millions)*	Percent Funded	Annual Revenue ($ millions)**	Ratio of Unfunded Liabilities to Revenues	Unfunded Liabilities per capita ($)***
New York	148,586	70	80,174	0.56	5,453
Los Angeles	35,063	89	8,854	0.44	1,017
Chicago	24,971	52	7,099	1.69	4,447
Philadelphia	18,337	62	6,382	1.09	4,566
San Francisco	17,257	97	6,043	0.09	643
Houston	11,030	80	3,456	0.64	1,051
Detroit	7,910	93	2,127	0.26	776
Dallas	7,359	87	2,497	0.38	799
San Diego	6,282	66	2,492	0.86	1,634
San Jose	5,450	79	1,646	0.70	1,210
Columbus	5,240	74	1,209	1.13	1,731
Phoenix	5,115	73	3,027	0.46	955
San Antonio	4,544	87	1,536	0.38	445
Jacksonville	4,028	63	1,993	0.75	1,814
Austin	3,729	75	1,304	0.71	1,179
Memphis	3,577	84	1,764	0.32	885
Fort Worth	2,301	81	997	0.44	590
El Paso	1,841	84	638	0.46	454
Charlotte	1,366	94	1,408	0.06	112
Indianapolis	1,162	94	2,399	0.03	84
Averages		79		0.57	1,492

*Data for liabilities and funding come from a report by the Pew Charitable Trusts (2013). The values are based on reported pension liabilities and assets in 2009 drawn from each municipality's Comprehensive Annual Financial Report.

**Data on municipal finances were taken from the Lincoln Institute of Land Policy's Fiscally Standardized Cities Database, 2010, available at www.lincolnist.edu. The values represent reported annual general revenues in 2010 dollars.

***Population data are taken from the 2010 U.S. Census.

the case in which the utility function takes the logarithmic form:

$$U = \ln c_{yt} + \psi \ln l_t + \beta \ln c_{ot+1}. \tag{42}$$

The fact that $v\left(c_o\right)$ is logarithmic implies that the demand for land with a binding down-payment constraint is independent of q_{t+1} and given by:

$$L(d_t, q_{t+1}) = \frac{\psi + \beta}{1 + \psi + \beta} \frac{w}{d_t}. \tag{43}$$

Sufficient conditions for the downpayment constraint to always be binding both in equilibrium and following any deviation are that: (i) the relative size of local government - as proxied by w^g and b^g - is sufficiently small and (ii) the share of land in utility is sufficiently large:[31]

$$\psi > \frac{\beta\left(R - 1\right)}{1 - \kappa}. \tag{44}$$

A high land share makes it more likely that the consumer is constrained because it increases his demand for land. The assumption about the size of government is needed because the existence of unfunded pension promises pushes taxes into the future, reducing land prices when an agent is old. As a consequence, agents might want to save rather than borrow. Assumption (i) limits the importance of this effect.

C   Proofs of Lemmas and Propositions

C.1   Proof of Proposition 1

(a) The function $U\left(w - d_t l, l, q_{t+1}\left(1 - \kappa\right) l\right)$ is continuous in l on the interval $[0, d_t/w]$. Therefore it achieves a maximum in this interval.

(b) The Inada conditions on the derivatives at $l = d_t/w$ and $l = 0$ rule out corner solutions in which the optimal demand for land is either d/w or zero. Therefore, the solution must be

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[31]The proof of this statement is contained in the Technical Appendix to the paper.

interior and satisfy the first-order condition (12):

$$-d_t u_1 \left(w - d_t l_t, l_t \right) + u_2 \left(w - d_t l_t, l_t \right) + v' \left(q_{t+1} l \left(1 - \kappa \right) \right) q_{t+1} \left(1 - \kappa \right) = 0. \qquad (45)$$

This equation admits a solution because of the Assumptions in part (b) of Proposition 1. Notice that the objective function $U \left(w - d_t l, l, q_{t+1} \left(1 - \kappa \right) l \right)$ is strictly concave in l_t because its second derivative with respect to l_t is negative:

$$\Delta = d_t^2 u_{11} \left(w - d_t l_t, l_t \right) - 2 d_t u_{12} \left(w - d_t l_t, l_t \right) + u_{22} \left(w - d_t l_t, l_t \right) + v'' \left(q_{t+1} l \left(1 - \kappa \right) \right) \left[q_{t+1} \left(1 - \kappa \right) \right]^2 < 0.$$

This is true because, by Assumption 1:

$$\begin{aligned} u_{11} \left(w - d_t l_t, l_t \right) &\leq 0, \\ u_{22} \left(w - d_t l_t, l_t \right) &\leq 0, \\ u_{12} \left(w - d_t l_t, l_t \right) &\geq 0, \\ v'' \left(q_{t+1} l \left(1 - \kappa \right) \right) &\leq 0, \end{aligned}$$

and at least one of the own-second derivatives is strictly negative. Thus, the solution to the first-order condition is unique.

(c) The properties of the demand function can be proved using the implicit function theorem as applied to equation (45). First:

$$\frac{\partial L \left(d_t, q_{t+1} \right)}{\partial d} = \frac{u_1 \left(w - d_t l_t, l_t \right) - d_t l_t u_{11} \left(w - d_t l_t, l_t \right) + u_{12} \left(w - d_t l_t, l_t \right) l_t}{\Delta} < 0$$

because the numerator of this expression is positive and the denominator is negative. The other derivative is:

$$\frac{dL \left(d_t, q_{t+1} \right)}{\partial q} = \frac{v' \left(q_{t+1} l \left(1 - \kappa \right) \right) + v'' \left(q_{t+1} l \left(1 - \kappa \right) \right) q_{t+1} l \left(1 - \kappa \right)}{-\Delta} \left(1 - \kappa \right)$$

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which can be expressed as:

$$\frac{\partial L\left(d_t, q_{t+1}\right)}{\partial q} = \frac{v'\left(q_{t+1}l\left(1-\kappa\right)\right)\left[1-\varepsilon\left(q_{t+1}l\left(1-\kappa\right)\right)\right]}{-\Delta}\left(1-\kappa\right), \tag{46}$$

where
$$\varepsilon\left(q_{t+1}l\left(1-\kappa\right)\right) \equiv -\frac{v''\left(q_{t+1}l\left(1-\kappa\right)\right)q_{t+1}l\left(1-\kappa\right)}{v'\left(q_{t+1}l\left(1-\kappa\right)\right)} > 0$$

is the (positive) elasticity of $v'(c_{ot+1})$ with respect to c_{ot+1}. It follows that the derivative (46) is negative if and only if $\varepsilon\left(q_{t+1}l\left(1-\kappa\right)\right) > 1$. In terms of κ, we can write:

$$\frac{\partial L\left(d_t, q_{t+1}\right)}{\partial \kappa} = \frac{\partial L\left(d_t, q_{t+1}\right)}{\partial d_t}\frac{\partial d_t}{\partial \kappa} + \frac{v'\left(q_{t+1}l\left(1-\kappa\right)\right)+v''\left(q_{t+1}l\left(1-\kappa\right)\right)q_{t+1}l\left(1-\kappa\right)}{\Delta}q_{t+1}.$$

Replacing $\partial L\left(d_t, q_{t+1}\right)/\partial d_t$ from above and noticing that $\partial d_t/\partial \kappa = -q_{t+1}/R$, leads to

$$
\begin{aligned}
&\frac{\partial L\left(d_t, q_{t+1}\right)}{\partial \kappa} \\
&= \frac{q_{t+1}}{R\left(-\Delta\right)}\left\{
\begin{array}{c}
u_1\left(w-d_tl_t, l_t\right) - d_tl_tu_{11}\left(w-d_tl_t, l_t\right) + u_{12}\left(w-d_tl_t, l_t\right)l_t - Rv'\left(q_{t+1}l\left(1-\kappa\right)\right) \\
-Rv''\left(q_{t+1}l\left(1-\kappa\right)\right)q_{t+1}l\left(1-\kappa\right)
\end{array}
\right\},
\end{aligned}
$$

where $\left(-\Delta\right) > 0$. The term in brackets is also positive because the agent is constrained and so $u_1\left(w-d_tl_t, l_t\right) > Rv'\left(q_{t+1}l\left(1-\kappa\right)\right)$.

(d) The indirect utility function is

$$V\left(d_t, q_{t+1}\right) = u\left(w-d_tL\left(d_t, q_{t+1}\right), L\left(d_t, q_{t+1}\right)\right) + v\left(q_{t+1}L\left(d_t, q_{t+1}\right)\left(1-\kappa\right)\right). \tag{47}$$

By the envelope theorem:

$$
\begin{aligned}
\frac{\partial V\left(d_t, q_{t+1}\right)}{\partial d_t} &= -u_1\left(w-d_tL\left(d_t, q_{t+1}\right), L\left(d_t, q_{t+1}\right)\right)L\left(d_t, q_{t+1}\right) < 0, \\
\frac{\partial V\left(d_t, q_{t+1}\right)}{\partial q_{t+1}} &= v'\left(q_{t+1}L\left(d_t, q_{t+1}\right)\left(1-\kappa\right)\right)L\left(d_t, q_{t+1}\right)\left(1-\kappa\right) > 0.
\end{aligned}
$$

Q.E.D.

C.2 Proof of Proposition 3

Uniqueness. Let z^* be defined as:

$$z^* \equiv \frac{1}{1 - \kappa/R} \left[w^g + \frac{f^* b^g}{R} + b^g (1 - f^*) \right], \tag{48}$$

so that equation (27) can be written as:

$$Q^* = \frac{D^*}{1 - \kappa/R} - z^*.$$

Replace Q^* into equation (28) to obtain:

$$P \left(V \left(D^*, \frac{D^*}{1 - \kappa/R} - z^* \right) \right) L \left(D^*, \frac{D^*}{1 - \kappa/R} - z^* \right) = 1. \tag{49}$$

Assume that there exists at least a D^* that satisfies equation (49). To show uniqueness of the solution we proceed in two steps. In the first one we show that the $L \left(D^*, \frac{D^*}{1-\kappa/R} - z^* \right)$ is a decreasing function of D^*. In the second step we show that $V \left(D^*, \frac{D^*}{1-\kappa/R} - z^* \right)$ is a decreasing function of D^*.

Step 1. Consider the first step and take the total derivative of L with respect to D^* :

$$\frac{\partial L (D^*, Q^*)}{\partial d} + \frac{\partial L (D^*, Q^*)}{\partial q} \frac{1}{1 - \kappa/R}.$$

This is negative if and only if:

$$\frac{\partial L (D^*, Q^*) / \partial q}{\partial L (D^*, Q^*) / \partial d} > - (1 - \kappa/R). \tag{50}$$

For convenience, define $L^* = L (D^*, Q^*)$. Use the implicit function theorem on equation

(12) to replace the derivatives and obtain:

$$\frac{\partial L\left(D^*, Q^*\right)/\partial q}{\partial L\left(D^*, Q^*\right)/\partial d}$$

$$= -\frac{v''\left(Q^*\left(1-\kappa\right)L^*\right)Q^*L^*(1-\kappa) + v'\left(Q^*\left(1-\kappa\right)L^*\right)}{u_1\left(w - D^*L^*, 1\right) - D^*L^*u_{11}\left(w - D^*L^*, 1\right) + L^*u_{12}\left(w - D^*L^*, 1\right)}(1-\kappa).$$

Notice that:

$$-\frac{v''\left(Q^*\left(1-\kappa\right)L^*\right)Q^*L^*(1-\kappa) + v'\left(Q^*\left(1-\kappa\right)L^*\right)}{u_1\left(w - D^*L^*, 1\right) - D^*L^*u_{11}\left(w - D^*L^*, 1\right) + L^*u_{12}\left(w - D^*L^*, 1\right)} > -\frac{v'\left(Q^*\left(1-\kappa\right)L^*\right)}{u_1\left(w - D^*L^*, 1\right)}$$

because $u_{12}\left(w - D^*L^*, 1\right) \geq 0$. Thus, it follows that:

$$\frac{\partial L\left(D^*, Q^*\right)/\partial q}{\partial L\left(D^*, Q^*\right)/\partial d} > -\frac{v'\left(Q^*\left(1-\kappa\right)L^*\right)}{u_1\left(w - D^*L^*, 1\right)}(1-\kappa) > -\frac{1-\kappa}{R} > -\left(1 - \frac{\kappa}{R}\right), \qquad (51)$$

where the second inequality holds due to the binding downpayment constraint and the third one holds because $R > 1$. This proves that equation (50) holds.

Step 2. The second step is to show that $V\left(D^*, \frac{D^*}{1-\kappa/R} - z^*\right)$ is a decreasing function of D^*. Take the total derivative of the indirect utility function:

$$\frac{\partial V\left(D^*, Q^*\right)}{\partial d} + \frac{\partial V\left(D^*, Q^*\right)}{\partial q}\frac{1}{1 - \kappa/R}.$$

This is negative if

$$\frac{\partial V\left(D^*, Q^*\right)/\partial q}{\partial V\left(D^*, Q^*\right)/\partial d} > -\left(1 - \kappa/R\right).$$

Apply the implicit function theorem to equation (47) to obtain:

$$\frac{\partial V\left(D^*, Q^*\right)/\partial q}{\partial V\left(D^*, Q^*\right)/\partial d} = -\frac{v'\left(Q^*\left(1-\kappa\right)L^*\right)}{u_1\left(w - D^*L^*, 1\right)}(1-\kappa).$$

The argument is then the same as in Step 1 of the proof - see equation (51). This proves the claim that $V\left(D^*, \frac{D^*}{1-\kappa/R} - z^*\right)$ is a decreasing function of D^*.

Since both $L\left(D^*, Q^*\right)$ and $V\left(D^*, \frac{D^*}{1-\kappa/R} - z^*\right)$ are monotonically decreasing in D^*, the

left-hand side of equation (49) is also monotonically decreasing with respect to D^*, guaranteeing that the solution is unique, provided it exists.

Existence. To show existence, notice first that the left-hand side of equation (49) is a continuous and monotonically decreasing function of D^*. Proving existence of equilibrium requires one to show that there exists a D^* that solves equation (49) above for a given $f^* \in [0, 1]$. We consider the case $w^g = b^g = 0$, which implies $z^* = 0$ in equation (48), and argue that the results that follow extend to the case $w^g, b^g > 0$ by continuity as long as w^g, b^g are not too large. Taking $w^g = b^g = 0$ into account, equation (49) becomes:

$$P\left(V\left(D^*, \frac{D^*}{1 - \kappa/R}\right)\right) L\left(D^*, \frac{D^*}{1 - \kappa/R}\right) = 1. \tag{52}$$

The strategy is to show that the function $L\left(D^*, \frac{D^*}{1-\kappa/R}\right)$ has the following properties:

$$\lim_{D^* \to 0} L\left(D^*, \frac{D^*}{1 - \kappa/R}\right) = +\infty \tag{53}$$

$$\lim_{D^* \to \infty} L\left(D^*, \frac{D^*}{1 - \kappa/R}\right) = 0. \tag{54}$$

If these two conditions hold, the left-hand side of equation (52) would have the same properties because the function $P(V)$ is bounded. These two properties are sufficient to guarantee that there exists a D^* such that the labor market is in equilibrium. Consider equation (53) and the first-order condition (12):

$$-D^* u_1\left(w - D^* L, L\right) + u_2\left(w - D^* L, L\right) + v'\left(\frac{D^*(1 - \kappa) L}{1 - \kappa/R}\right) \frac{D^*(1 - \kappa)}{1 - \kappa/R} = 0. \tag{55}$$

As $D^* \to 0$, the demand for land grows without bound because the marginal cost of land

$$D^* u_1\left(w - D^* L, L\right) \to 0$$

and the marginal benefit of land is always strictly positive by assumption (1):

$$u_2\left(w - D^*L, L\right) + v'\left(\frac{D^*\left(1 - \kappa\right)L}{1 - \kappa/R}\right)\frac{D^*\left(1 - \kappa\right)}{1 - \kappa/R} > 0.$$

Consider now equation (54). When $D^* > 0$, equation (55) can be re-written as:

$$-u_1\left(w - D^*L, L\right) + \frac{u_2\left(w - D^*L, L\right)}{D^*} + v'\left(\frac{D^*\left(1 - \kappa\right)L}{1 - \kappa/R}\right)\frac{1 - \kappa}{1 - \kappa/R} = 0.$$

As $D^* \to +\infty$, consumption when young goes to zero and the marginal cost of land becomes arbitrarily large:

$$-u_1\left(w - D^*L, L\right) \to -\infty.$$

The marginal benefit of land, instead declines because $u_{21}\left(w - D^*L, L\right) \geq 0$ and the function $v'\left(.\right)$ is weakly decreasing in its argument. It follows that the demand for land must converge to zero as $D^* \to +\infty$. It follows that the demand for land on the left-hand side of (52) must intersect supply for some D^*. The same logic can be applied, by continuity, when $w^g, b^g > 0$ as long as w^g, b^g are not too large. Q.E.D.

C.3   Proof of Lemma 1

To determine the derivative of $D\left(\widetilde{f}'; f^*\right)$ with respect to $\widetilde{f}'$, apply the implicit function theorem to equation (29), taking into account the fact that

$$Q\left(\widetilde{f}'; f^*\right) = \frac{D\left(f^*; f^*\right) - \left[w^g + b^g\left(\left(1 - \kappa\right)f^* + \kappa\right)/R\right]}{1 - \kappa/R} - b^g\left(1 - \widetilde{f}'\right), \tag{56}$$

and therefore:

$$\frac{\partial Q\left(\widetilde{f}'; f^*\right)}{\partial \widetilde{f}'} = b^g. \tag{57}$$

From the implicit function theorem we obtain that:

$$\frac{\partial D\left(\widetilde{f}';f^*\right)}{\partial \widetilde{f}'} = \frac{\frac{1}{L^2(D,Q)}L_2\left(D,Q\right)b^g + P'\left(V\right)V_2\left(D,Q\right)b^g}{-\frac{1}{L^2(D,Q)}L_1\left(D,Q\right) - P'\left(V\right)V_1\left(D,Q\right)}$$

where D and Q on the right-hand side of this equation stand for $D\left(\widetilde{f}';f^*\right)$ and $Q\left(\widetilde{f}';f^*\right)$. We can write the term:

$$\frac{1}{b^g}\frac{\partial D\left(\widetilde{f}';f^*\right)}{\partial \widetilde{f}'}$$

$$= \frac{\frac{1}{L^2(D,Q)}L_2\left(D,Q\right) + P'\left(V\right)V_2\left(D,Q\right)}{-\frac{1}{L^2(D,Q)}L_1\left(D,Q\right) - P'\left(V\right)V_1\left(D,Q\right)}$$

$$= \frac{\frac{1}{L^2(D,Q)}L_2\left(D,Q\right)}{-\frac{1}{L^2(D,Q)}L_1\left(D,Q\right) - P'\left(V\right)V_1\left(D,Q\right)} + \frac{P'\left(V\right)V_2\left(D,Q\right)}{-\frac{1}{L^2(D,Q)}L_1\left(D,Q\right) - P'\left(V\right)V_1\left(D,Q\right)}$$

$$= \frac{\frac{1}{L^2(D,Q)}}{\frac{1}{L^2(D,Q)} + P'\left(V\right)V_1\left(D,Q\right)/L_1\left(D,Q\right)}\left(\frac{L_2\left(D,Q\right)}{-L_1\left(D,Q\right)}\right)$$

$$+ \frac{P'\left(V\right)}{\frac{1}{L^2(D,Q)}L_1\left(D,Q\right)/V_1\left(D,Q\right) + P'\left(V\right)}\left(\frac{V_2\left(D,Q\right)}{-V_1\left(D,Q\right)}\right)$$

$$= \omega\left(D,Q\right)\left(-\frac{L_2\left(D,Q\right)}{L_1\left(D,Q\right)}\right) + \left(1 - \omega\left(D,Q\right)\right)\left(-\frac{V_2\left(D,Q\right)}{V_1\left(D,Q\right)}\right), \tag{58}$$

where the "weight" $\omega\left(D,Q\right)$ is defined as:

$$\omega\left(D,Q\right) \equiv \frac{1}{1 + L^2\left(D,Q\right)P'\left(V\right)V_1\left(D,Q\right)/L_1\left(D,Q\right)}. \tag{59}$$

The term $\left(-V_2\left(D,Q\right)/V_1\left(D,Q\right)\right)$ can be found considering the indirect utility of a young agent

$$V\left(D,Q\right) = U(w - DL\left(D,Q\right), L\left(D,Q\right), Q\left(1 - \kappa\right)L\left(D,Q\right))) \tag{60}$$

and apply the envelope theorem to obtain the partial derivatives:

$$V_1(D, Q) = -u_1(w - DL, L)L \tag{61}$$

$$V_2(D, Q) = v'(Q(1 - \kappa)L)(1 - \kappa)L, \tag{62}$$

where L denotes $L(D, Q)$. We therefore obtain that:

$$-\frac{V_2(D, Q)}{V_1(D, Q)} = \frac{v'(Q(1 - \kappa)L)}{u_1(w - DL, L)}(1 - \kappa). \tag{63}$$

Moreover, it is straightforward to show using the implicit function theorem on the first-order condition for land (equation 12) that:

$$-\frac{L_2(D, Q)}{L_1(D, Q)} = \frac{v''(Q(1 - \kappa)L)Q(1 - \kappa)L + v'(Q(1 - \kappa)L)}{u_1(w - DL, L) - LDu_{11}(w - D, L) + Lu_{12}(w - DL, L)}(1 - \kappa). \tag{64}$$

Notice that

$$-\frac{V_2(D, Q)}{V_1(D, Q)} \geq -\frac{L_2(D, Q)}{L_1(D, Q)} \tag{65}$$

because by Assumption 1:

$$v''(Q(1 - \kappa)L) \leq 0, \tag{66}$$

$$u_{11}(w - D, L) \leq 0, \tag{67}$$

$$u_{12}(w - DL, L) \geq 0. \tag{68}$$

It follows from equation (65) and the fact that $\omega(D, Q) \in [0, 1]$ that:

$$\frac{1}{b^g}\frac{\partial D(\tilde{f}'; f^*)}{\partial \tilde{f}'} \leq -\frac{V_2(D, Q)}{V_1(D, Q)}. \tag{69}$$

Replacing equation (63) in equation (69) and rearranging gives the desired result, i.e. equation (30). The inequality in this equation is strict whenever the weight $\omega(D, Q)$ is strictly greater than zero and at least one of the inequalities (66)-(68) is strict. The condition

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that $\omega\left(D,Q\right)>0$ requires $P'\left(V\right)<+\infty$ (see equation (59)).

Q.E.D.

C.4 Proof of Proposition 4

(1) **Effect on the land price today**. We want to show that:

$$\frac{\partial\widetilde{Q}\left(f,\widetilde{f}';f^*\right)}{\partial\widetilde{f}'}=\frac{\partial D\left(\widetilde{f}';f^*\right)}{\partial\widetilde{f}'}+\frac{\left(\kappa-1\right)b^g}{R}<0, \tag{70}$$

where the upper bound on $\partial D\left(\widetilde{f}';f^*\right)/\partial\widetilde{f}'$ has been established in Lemma 1. It is then sufficient to show that the term

$$-\frac{V_2\left(D,Q\right)}{V_1\left(D,Q\right)}+\frac{\kappa-1}{R}<0 \tag{71}$$

for (70) to hold. This is true because of equation (63) and because the downpayment constraint is binding:

$$\frac{v'(Q\left(1-\kappa\right)L)}{u_1(w-DL,L)}<\frac{1}{R}.$$

(2) **Effect on lifetime utility**. Notice that:

$$\begin{aligned}\frac{\partial V\left(D\left(\widetilde{f}';f^*\right),Q\left(\widetilde{f}';f^*\right)\right)}{\partial\widetilde{f}'}&=V_1\left(D,Q\right)\frac{\partial D\left(\widetilde{f}';f^*\right)}{\partial\widetilde{f}'}+V_2\left(D,Q\right)b^g\\&=b^gV_1\left(D,Q\right)\left[\frac{1}{b^g}\frac{\partial D\left(\widetilde{f}';f^*\right)}{\partial\widetilde{f}'}-\left(-\frac{V_2\left(D,Q\right)}{V_1\left(D,Q\right)}\right)\right].\end{aligned}$$

We want to show that the term in squared brackets is non-positive given that $V_1\left(D,Q\right)<0$. This is the case according to the inequality in equation (69). Moreover, if the utility function satisfies at least one of the inequalities (66)-(68), then the term in square brackets is strictly negative and so the derivative of $V\left(D\left(\widetilde{f}';f^*\right),Q\left(\widetilde{f}';f^*\right)\right)$ with respect to $\widetilde{f}'$ is

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strictly positive.

Q.E.D.

C.5 Proof of Proposition 5

As we write in the main text, when the borrowing constraint does not bind the demand for land depends only on its user cost. Thus, equation (25) holds with $\kappa = 1$. In that equation, D does not depend on $\widetilde{f}'$. The derivative of the future land price with respect to $\widetilde{f}'$ is simply:

$$\frac{\partial Q\left(\widetilde{f}'; f^*\right)}{\partial \widetilde{f}'} = b^g. \tag{72}$$

Thus, we compute:

$$\frac{\partial \widetilde{Q}\left(f, \widetilde{f}'; f^*\right)}{\partial \widetilde{f}'} = \frac{1}{R}\frac{\partial Q\left(\widetilde{f}'; f^*\right)}{\partial \widetilde{f}'} - \frac{b^g}{R} \tag{73}$$

which equals zero. Q.E.D.

C.6 Proof of Proposition 6

If the young set the policy and the assumptions in Corollary 1 hold, the equilibrium funding rule is $f^* = 1$, so $f_{\min}$ is irrelevant.

If the old set the policy in equilibrium we have:

$$f^* = f_{\min}. \tag{74}$$

Utility of initial old. The land price in equation (26) represents the utility of the first generation. From that from equation (26) we determine that $Q\left(f; f^*\right)$ decreases with f^* if and only if:

$$\frac{\partial D\left(f^*; f^*\right)}{\partial f^*} < \frac{b^g\left(1 - \kappa\right)}{R}. \tag{75}$$

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Take the total derivative of the labor market clearing condition:

$$P(V(D^*, Q^*))L(D^*, Q^*) = 1 \tag{76}$$

with respect to f^* and D^*. We obtain:

$$P'(V(D^*, Q^*))L(D^*, Q^*)\left[V_1(D^*, Q^*)\partial D(f^*; f^*) + V_2(D^*, Q^*)\frac{\partial D(f^*; f^*)}{1 - \kappa/R} - \frac{b^g/R - b^g}{1 - \kappa/R}V_2(D^*, Q^*)\partial f^*\right]$$
$$+ P(V(D^*, Q^*))\left[L_1(D^*, Q^*)\partial D(f^*; f^*) + L_2(D^*, Q^*)\frac{\partial D(f^*; f^*)}{1 - \kappa/R} - \frac{b^g/R - b^g}{1 - \kappa/R}L_2(D^*, Q^*)\partial f^*\right] = 0.$$

Solve the latter for derivative the derivative of interest:

$$\frac{\partial D(f^*; f^*)}{\partial f^*} = b^g\frac{1/R - 1}{1 - \kappa/R} \tag{77}$$

$$\frac{P'(V(D^*, Q^*))L(D^*, Q^*)V_2(D^*, Q^*) + P(V(D^*, Q^*))L_2(D^*, Q^*)}{P'(V(D^*, Q^*))L(D^*, Q^*)\left[V_1(D^*, Q^*) + \frac{V_2(D^*, Q^*)}{1 - \kappa/R}\right] + P(V(D^*, Q^*))\left[L_1(D^*, Q^*) + \frac{L_2(D^*, Q^*)}{1 - \kappa/R}\right]}$$
$$= -b^g(1 - 1/R) \times \text{TERM},$$

where

$$\text{TERM} \equiv \frac{P'(V(D^*, Q^*))V_2(D^*, Q^*) + L_2(D^*, Q^*)/(L^*)^2}{(1 - \kappa/R)\left[P'(V(D^*, Q^*))V_1(D^*, Q^*) + \frac{L_1(D^*, Q^*)}{(L^*)^2}\right] + P'(V(D^*, Q^*))V_2(D^*, Q^*) + \frac{L_2(D^*, Q^*)}{(L^*)^2}}. \tag{78}$$

Notice that we have used the fact that $P(V(D^*, Q^*)) = 1/L^*$ and that

$$P'(V(D^*, Q^*))V_1(D^*, Q^*) + \frac{L_1(D^*, Q^*)}{(L^*)^2} < 0,$$

because $V_1(D^*, Q^*) < 0$ and $L_1(D^*, Q^*) < 0$. Condition (75) then reduces to:

$$-(R - 1) \times \text{TERM} < (1 - \kappa). \tag{79}$$

There are two options, corresponding to whether TERM> 0 or TERM< 0. In case

TERM> 0, condition (79) is satisfied because $R > 1$. Consider then the case in which TERM< 0. For this to be true we need:

$$P'(V(D^*, Q^*))V_2(D^*, Q^*) + L_2(D^*, Q^*) / (L^*)^2 > 0, \tag{80}$$

otherwise TERM> 0. Divide both numerator and denominator of equation (78) by (80) to re-write TERM as:

$$\text{TERM} = \frac{1}{(1 - \kappa/R) \frac{P'(V(D^*,Q^*))V_1(D^*,Q^*) + \frac{L_1(D^*,Q^*)}{(L^*)^2}}{P'(V(D^*,Q^*))V_2(D^*,Q^*) + L_2(D^*,Q^*)/(L^*)^2} + 1}. \tag{81}$$

With TERM< 0, equation (79) becomes:

$$\frac{1}{-\text{TERM}} > \frac{R-1}{1-\kappa}, \tag{82}$$

with

$$\frac{1}{-\text{TERM}} = -(1 - \kappa/R) \frac{P'(V(D^*, Q^*))V_1(D^*, Q^*) + \frac{L_1(D^*,Q^*)}{(L^*)^2}}{P'(V(D^*, Q^*))V_2(D^*, Q^*) + L_2(D^*, Q^*) / (L^*)^2} - 1. \tag{83}$$

Replace this in (82) and simplify to obtain:

$$\frac{P'(V(D^*, Q^*)) [-V_1(D^*, Q^*)] + \frac{[-L_1(D^*,Q^*)]}{(L^*)^2}}{P'(V(D^*, Q^*))V_2(D^*, Q^*) + L_2(D^*, Q^*) / (L^*)^2} > \frac{R}{1-\kappa}. \tag{84}$$

Notice that the left-hand side of this inequality can be written as:

$$\eta^* \frac{[-V_1(D^*, Q^*)]}{V_2(D^*, Q^*)} + (1 - \eta^*) \frac{[-L_1(D^*, Q^*)]}{L_2(D^*, Q^*)},$$

where

$$\eta^* \equiv \frac{P'(V(D^*, Q^*))V_2(D^*, Q^*)}{P'(V(D^*, Q^*))V_2(D^*, Q^*) + L_2(D^*, Q^*) / (L^*)^2}.$$

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From equation (65) we know that:

$$\frac{[-V_1(D^*, Q^*)]}{V_2(D^*, Q^*)} \le \frac{[-L_1(D^*, Q^*)]}{L_2(D^*, Q^*)}. \tag{85}$$

Thus, we only need to show that

$$\frac{[-V_1(D^*, Q^*)]}{V_2(D^*, Q^*)} > \frac{R}{1-\kappa} \tag{86}$$

for (82) to hold. This is the case because of the bindind downpayment constraint as:

$$\frac{[-V_1(D^*, Q^*)]}{V_2(D^*, Q^*)} = \frac{v'(Q^*(1-\kappa)L(D^*, Q^*))}{u_1(w - D^*L(D^*, Q^*), L(D^*, Q^*))}(1-\kappa) < \frac{1-\kappa}{R}.$$

Lifetime utility of young. Consider now the lifetime utility of a young agent, $V(D(f^*; f^*), Q(f^*; f^*))$. We have that

$$\frac{\partial V(D(f^*; f^*), Q(f^*; f^*))}{\partial f^*} = V_1(D^*, Q^*)\frac{\partial D(f^*; f^*)}{\partial f^*} + V_2(D^*, Q^*)\frac{\partial Q(f^*; f^*)}{\partial f^*},$$

where D^* stands for $D(f^*; f^*)$, and Q^* stands for $Q(f^*; f^*)$. For this derivative to be positive we need:

$$\frac{\partial D(f^*; f^*)}{\partial f^*} + \frac{V_2(D^*, Q^*)}{V_1(D^*, Q^*)}\frac{\partial Q(f^*; f^*)}{\partial f^*} < 0. \tag{87}$$

From equation (27), we know that:

$$\frac{\partial Q(f^*; f^*)}{\partial f^*} = \frac{1}{1 - \kappa/R}\frac{\partial D(f^*; f^*)}{\partial f^*} + \frac{b^g(1 - 1/R)}{1 - \kappa/R}. \tag{88}$$

Replacing this into equation (87), we need to show that:

$$\frac{\partial D(f^*; f^*)}{\partial f^*} + \frac{1}{1 - \kappa/R}\frac{V_2(D^*, Q^*)}{V_1(D^*, Q^*)}\left[\frac{\partial D(f^*; f^*)}{\partial f^*} + b^g(1 - 1/R)\right] < 0.$$

Replace equation (77):

$$-\text{TERM} \times b^g \left(1 - 1/R\right) \left\{1 + \frac{1}{1 - \kappa/R} \frac{V_2\left(D^*, Q^*\right)}{V_1\left(D^*, Q^*\right)}\right\} + \frac{b^g\left(1 - 1/R\right)}{1 - \kappa/R} \frac{V_2\left(D^*, Q^*\right)}{V_1\left(D^*, Q^*\right)} < 0.$$

Simplify:

$$-\text{TERM} \times \left\{1 - \kappa/R - \frac{V_2\left(D^*, Q^*\right)}{\left[-V_1\left(D^*, Q^*\right)\right]}\right\} - \frac{V_2\left(D^*, Q^*\right)}{\left[-V_1\left(D^*, Q^*\right)\right]} < 0. \tag{89}$$

If TERM≥ 0 then (89) is satisfied because $V_2\left(D^*, Q^*\right)/V_1\left(D^*, Q^*\right) < 0$ and

$$1 - \frac{\kappa}{R} - \frac{V_2\left(D^*, Q^*\right)}{\left[-V_1\left(D^*, Q^*\right)\right]} > \frac{1}{R} - \frac{\kappa}{R} - \frac{V_2\left(D^*, Q^*\right)}{\left[-V_1\left(D^*, Q^*\right)\right]} > 0$$

due to equation (86) and $R > 1$. Assume then that TERM< 0. Rearrange equation (89) to read:

$$\frac{1}{-\text{TERM}} > \frac{1 - \kappa/R}{\frac{V_2\left(D^*, Q^*\right)}{\left[-V_1\left(D^*, Q^*\right)\right]}} - 1.$$

Replace equation (83) and simplify:

$$\frac{P'\left(V\left(D^*, Q^*\right)\right)\left[-V_1\left(D^*, Q^*\right)\right] + \frac{\left[-L_1\left(D^*, Q^*\right)\right]}{\left(L^*\right)^2}}{P'\left(V\left(D^*, Q^*\right)\right)V_2\left(D^*, Q^*\right) + L_2\left(D^*, Q^*\right)/\left(L^*\right)^2} > -\frac{V_1\left(D^*, Q^*\right)}{V_2\left(D^*, Q^*\right)}.$$

Using the expression for the left-hand side derived above (equation (84)) we obtain:

$$\eta^* \frac{\left[-V_1\left(D^*, Q^*\right)\right]}{V_2\left(D^*, Q^*\right)} + \left(1 - \eta^*\right) \frac{\left[-L_1\left(D^*, Q^*\right)\right]}{L_2\left(D^*, Q^*\right)} > -\frac{V_1\left(D^*, Q^*\right)}{V_2\left(D^*, Q^*\right)}.$$

From equation (65) we know that this inequality is always satisfied weakly and strictly if the utility function satisfies at least one of the inequalities (66)-(68).

Future price of land. We show that the future price of land $Q\left(f^*; f^*\right)$ is increasing in f^*. From equation (27), this is the case if and only if:

$$\frac{\partial D\left(f^*; f^*\right)}{\partial f^*} + b^g\left(1 - 1/R\right) > 0. \tag{90}$$

Use equation (77) to replace $\partial D\left(f^*; f^*\right)/\partial f^*$ and re-write (90) as:

$$-b^g\left(1-1/R\right)\times\text{TERM}+b^g\left(1-1/R\right)>0.$$

Simplify this equation to:

$$\text{TERM}<1.$$

Use equation (78) to replace TERM:

$$\frac{\frac{P'(V(D^*,Q^*))V_2(D^*,Q^*)+L_2(D^*,Q^*)/(L^*)^2}{P'(V(D^*,Q^*))V_1(D^*,Q^*)+\frac{L_1(D^*,Q^*)}{(L^*)^2}}}{1-\kappa/R+\frac{P'(V(D^*,Q^*))V_2(D^*,Q^*)+\frac{L_2(D^*,Q^*)}{(L^*)^2}}{P'(V(D^*,Q^*))V_1(D^*,Q^*)+\frac{L_1(D^*,Q^*)}{(L^*)^2}}}<1. \tag{91}$$

There are two cases. First, if

$$\frac{P'(V\left(D^*,Q^*\right))V_2\left(D^*,Q^*\right)+L_2\left(D^*,Q^*\right)/\left(L^*\right)^2}{P'(V\left(D^*,Q^*\right))V_1\left(D^*,Q^*\right)+\frac{L_1(D^*,Q^*)}{(L^*)^2}}>0, \tag{92}$$

then equation (91) is verified. Second, if the inequality in (92) is not verified, then (91) is verified as long as

$$1-\kappa/R+\frac{P'(V\left(D^*,Q^*\right))V_2\left(D^*,Q^*\right)+\frac{L_2(D^*,Q^*)}{(L^*)^2}}{P'(V\left(D^*,Q^*\right))V_1\left(D^*,Q^*\right)+\frac{L_1(D^*,Q^*)}{(L^*)^2}}>0.$$

To show that this is the case, recall that

$$\frac{P'(V\left(D^*,Q^*\right))V_2\left(D^*,Q^*\right)+\frac{L_2(D^*,Q^*)}{(L^*)^2}}{P'(V\left(D^*,Q^*\right))V_1\left(D^*,Q^*\right)+\frac{L_1(D^*,Q^*)}{(L^*)^2}}=-\left\{\phi^*\frac{V_2\left(D^*,Q^*\right)}{\left[-V_1\left(D^*,Q^*\right)\right]}+\left(1-\phi^*\right)\frac{L_2\left(D^*,Q^*\right)}{\left[-L_1\left(D^*,Q^*\right)\right]}\right\},$$

where $\phi^*\in(0,1)$ is defined as:

$$\phi^*\equiv\frac{P'(V\left(D^*,Q^*\right))\left[-V_1\left(D^*,Q^*\right)\right]}{P'(V\left(D^*,Q^*\right))\left[-V_1\left(D^*,Q^*\right)\right]+\frac{\left[-L_1(D^*,Q^*)\right]}{(L^*)^2}}.$$

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Notice then that:

$$1 - \frac{\kappa}{R} > \frac{1 - \kappa}{R} > \frac{V_2\left(D^*, Q^*\right)}{\left[-V_1\left(D^*, Q^*\right)\right]} \geq \phi^* \frac{V_2\left(D^*, Q^*\right)}{\left[-V_1\left(D^*, Q^*\right)\right]} + \left(1 - \phi^*\right) \frac{L_2\left(D^*, Q^*\right)}{\left[-L_1\left(D^*, Q^*\right)\right]}$$

where the first inequality follows from $R > 1$, the second one from equation (71), and the third one from (65).

Q.E.D.