The Production Function for Housing: Evidence from France

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Abstract: We propose a new non-parametric approach to estimate the production function for housing. Our estimation treats output as a latent variable and relies on the first-order condition for house builders. For parcels of similar sizes, we compute housing by summing across the marginal products of non-land inputs. In turn, differences in non-land inputs are caused by differences in land prices that reflect differences in the demand for housing across locations. We implement our methodology on recently-built single-family homes in France. We find that the production function for housing is reasonably well approximated by a Cobb-Douglas function and close to constant returns. After correcting for differences in user costs between different inputs, we obtain a share of land of about 20% in the production of housing.

Key words: housing, production function.

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1. Introduction

We propose a new non-parametric approach to estimate the production function for housing. Our estimation treats output as a latent variable and relies on the first-order condition for house builders. For parcels of similar sizes, we compute housing by summing across the marginal products of non-land inputs. In turn, differences in non-land inputs are caused by differences in land prices that reflect differences in the demand for housing across locations. We implement our methodology on recently-built single-family homes in France. We find that the production function for housing is reasonably well approximated by a Cobb-Douglas function and close to constant returns. After correcting for differences in user costs between different inputs, we obtain a share of land of about 20% in the production of housing.

A good understanding of the supply of housing is important for a number of reasons. First, housing is an unusually important good. It arguably provides an essential service to households and represents around 25% of their expenditure in both the US and France (Davis and Ortalo-Magné, 2011, Commissariat Général au Développement Durable, 2011). It is also an unusually important asset. The value of the US residential stock owned by households was around 20 trillion dollar in 2007 (Gyourko, 2009). French households owned about 4.6 trillion dollar worth of housing in 2011 (Mauro, 2013). For both countries, this represents about 180% of their gross domestic product.

Housing and the construction industry also matter to the broader economy. The construction industry, which in France employed about 8% of the workforce in 2011, is arguably an important driver of the business cycle (e.g., Davis and Heathcote, 2005). The role of housing in the great recession has been studied by, among others, Chatterjee and Eyigungor (2011) and Kiyotaki, Michaelides, and Nikolov (2011). The broader effects of housing are not limited to the business cycle. Housing has also been argued to affect a variety of aggregate variables such as unemployment (Head and Lloyd-Ellis, 2012, Rupert and Wasmer, 2012) or economic growth (Davis, Fisher, and Whited, 2011).

Finally, and most importantly, housing is also central to our understanding of cities. Different locations within a city offer different levels of accessibility and bundles of amenities. Housing production is central in transforming the demand for locations from households into patterns of land use and housing consumption. Unsurprisingly, housing is at the heart of land use models in the spirit of Alonso (1964), Muth (1969), and Mills (1967) that form the core of modern urban economics. The exact shape of the housing production function has important implications for the welfare properties of a range of planning regulations (Larson and Yezer, 2013). In particular, the welfare consequences of extent land use restrictions such as minimum lot size limits will depend on how substitutable land is in the production of housing.

Following Muth's (1969, 1975) pioneering work, there is a long tradition of work estimating a production function for housing. The early literature is reviewed in McDonald (1981). A standard approach is to regress some measure of property prices on the values of land and other inputs in a cost-share estimation of a Cobb-Douglas production function.[1] A closely related alternative is, like Albouy and Ehrlich (2012), to estimate a cost function using land price data and a construction costs index.

More generally, the estimation of the production function for housing is traditionally conducted in the same way we estimate production functions or cost functions for other goods in productivity studies (see Ackerberg, Benkard, Berry, and Pakes, 2007, Syverson, 2011, for recent reviews of the literature estimating production functions).[2] Results are generally supportive of constant returns to scale in the production of housing and estimates for the elasticity of substitution between land and other inputs typically range between 0.50 and 0.75.

Traditional approaches to the estimation of the production function for housing suffer from a number of problems. First, housing is highly heterogeneous. Any given residential property is essentially unique due to the uniqueness of locations. Properties also differ by their size, quality, and a range of other characteristics such as their partition into room, their architectural style, or their level of maintenance. As a result, when a property is transacted, we can observe the value of the transaction $P \times H$ but not the true quantity of housing being traded H nor the price per unit P. This is a version of the unobserved price / unobserved quality problem that usually plagues the

[1] To estimate a more general constant-elasticity-of-substitution production function, past research has often regressed the capital to land ratio of properties on the price of of their land. This directly estimates the elasticity of substitution between capital and land in the production of housing. However, as it involves regressing ratios of quantities on prices instead of prices on quantities, this type of estimation is, if anything, more problematic than the estimation of the production function.

[2] Epple, Gordon, and Sieg (2010) is an exception. We discuss their approach in greater details below after sketching our own approach.

estimation of production functions. This problem is particularly acute with housing.[3] Within a large urban area, the price of a square meter of housing easily varies by a full order of magnitude.[4] The difficulty to separate prices from quantities also matters more with housing. In many production function contexts, knowing how much revenue firms can generate from inputs is often something we care deeply about whereas, for housing, actually knowing how many units of housing are supplied is of utmost importance.

Then, we also acknowledge some parcels may be more suitable for housing production in a way that is not observable to the analyst. To draw a parallel with the standard estimation of production function, some parcels may be more productive than others. As a result, more housing will be built on more productive parcels and, in turn, this will affect input choices. This is the classic endogeneity problem that plagues the estimation of production functions. The added complication is that, with housing, the suitability of a parcel for construction will also be capitalised in its price.

To summarise, the specificities of housing call for specific estimation techniques, impose strong data constraints, and require careful attention to the sources of variation used for identification.

To meet our first challenge and be able to separate the quantity of housing H from its price P, we develop a novel approach. This approach relies on three main assumptions. First and despite their extreme heterogeneity, we assume that houses can be compared in terms of how much housing services H they offer. Put differently, there is a production function for housing $H(K{,}T)$, which uses land T and capital (i.e., non-land inputs) K as primary factors.[5] Since it cannot be directly observed, the quantity of housing is best thought of as a latent variable. Second, house builders maximise profit. They choose how much capital to use in order to build a house on a particular parcel of land given the price P that households are willing to pay for each unit of housing on

[3]A natural approach for housing economists would be to purge the price of housing per unit of built area of the effect of observable characteristics through an hedonic approach. This amounts to defining arbitrarily what a unit of H is (e.g., a squarefoot of floorspace in a certain type of house) before getting P and in turn using it to recover H. Beyond being extremely indirect and roundabout, such procedure is also poorly identified since we expect observed and unobserved characteristics to be correlated with the true price per unit P. Two other solutions have been considered by researchers in industrial organisation to tackle the issues of unobserved price/quality. The first is to focus the estimation on truly homogenous goods (see for instance, Syverson, 2004, who restricts attention to firms in the ready-made concrete sector). This would be extremely limiting with housing since one would need to restrict attention to something like mobile homes. Second, and more indirectly, one may want to estimate the elasticity of housing production with respect to input prices. A standard strategy is to instrument prices in a location by prices in other locations (as for instance Nevo, 2001). While the appropriateness of this strategy is debatable for consumption goods produced by oligopolistic firms, it is impossible it to implement in the housing market where land is immobile.

[4]According to the website Meilleurs Agents (`http://www.meilleursagents.com/`), the ratio between the price per square meter at the higher bracket of the most expensive part of Paris and that at the lower brackets in the poorer northeastern suburbs is about 10.

[5]The notion of a production for housing services can be traced back to Muth (1960) and Olsen (1969).

this parcel. Third, we also assume free entry among builders. This is a fair assumption for the construction of single-family homes as we argue below.

When solving our model, the first-order condition for profit maximisation by house builders implies that the price of housing per unit times the marginal product of capital should be equal to the user cost of capital: $P\,H_K = r$. Under free entry, the difference between the price of a house and the cost of the capital used to produce it should be equal to the cost of the parcel: $P\,H - rK = R(T)$. We can use the free-entry condition to eliminate the price of housing from the first-order condition for profit maximisation and obtain a partial differential equation where the marginal product of capital depends on the quantity of housing produced and the cost and quantity of both factors: $H_K = r/P = r\,H/(rK + R(T))$. For any given amount of land T, this partial differential equation can be solved up to a constant. The solution involves integrating over H_K/H. This allows us to estimate the amount of housing H built on a parcel using information about the cost of capital, its amount, and the cost of land for the parcel at hand and for the other parcels of the same area T built with less capital. In turn, knowing about housing H and capital K, we can assess how the quantity of housing varies with capital for a given land area T.

Because our estimation is conditional on land area T, the production function for housing is only partially identified. More specifically, it is only identified for a particular quantile of land area. However, we can compare across quantiles of land area and verify that we obtain similar results. We note that our methodology, although particularly appropriate for housing, is not specific to it. It could be applied to estimating the production function of any good for which there is (enough) variation in the price of a key input.

The second challenge is to find appropriate data. Our methodology requires information about the price of parcels, their land area, and the amount of capital used for construction. The unique data we use satisfy these requirements. They consist of several large annual cross-sections of land parcels sold in France with a building permit for a single-family home.

Given our approach and the data at hand, the third challenge is to use an appropriate source of variation. As already argued, the price of land may reflect not only the demand for housing locally but also some supply factors. Although our estimation technique is non-standard, we still face the same identification issue as standard regression estimation of the housing production function. The supply of housing can only be identified from variation in the demand for housing, not from unobserved differences in supply conditions. To minimise this problem, we develop an instrumental variable type of approach and use the fact that the demand for housing varies systematically both across cities and within cities depending on the relative location of a parcel. Housing located closer to the centre of Paris is more expensive than housing located further away in the suburbs. Such difference is more plausibly caused by difference in demand rather than by differences in the ease of construction.

We obtain three main results. First, we find that the share of land in housing is roughly constant at 20%. As a first-order approximation, housing is produced under constant returns to scale and is Cobb-Douglas in land and capital. This said, we can nonetheless formally reject that the housing production function is Cobb-Douglas and constant returns. We can also reject more general functional forms such as the translog or the CES. We find evidence of a slight complementarity between land and capital and of small decreasing returns.

Finally, our work is most closely related to Epple *et al.* (2010) and Ahlfeldt and McMillen (2013) who also treat housing as a latent variable.[6] Like us, they provide a non-parametric estimation of the housing production function using restrictions from theory. We nonetheless differ from their approach in several key respects. Unlike us, they assume constant returns to scale. For each unit of land, this assumption allows them to express the first-order condition for profit maximisation with respect to capital in terms of the unit price of housing. The latter is not observed but they show that it can be constructed as a monotonic function of the value of housing per unit of land. Our approach shows that imposing constant returns to scale is unwarranted. They also rely on different observables, namely housing values per unit of land PH/T and land rent per unit of land R/T instead of land rent and capital for each quantile of land area. The use of land rent per unit of land may be problematic since this price is highly correlated with parcel area in our data. We also implement our approach on very different data: newly constructed houses for an entire country instead of assessed land values for all houses for a single city, Pittsburgh, as Epple *et al.* (2010). Finally, we take steps towards disentangling demand and supply factors in land prices, an issue ignored by Epple *et al.* (2010).[7]

[6] Gandhi, Navarro, and Rivers (2013) also estimate production functions non-parametrically using a first-order condition for profit maximisation but their approach focuses on the simultaneity between input choices and output rather than disentangling prices and quantities. We treat the simultaneity of inputs differently through an instrumental variable approach.

[7] These differences notwithstanding, our results are broadly consistent with theirs and supportive of unitary elasticity of substitution between land and capital. They find a lower coefficient for land of 0.14 instead of 0.20. We attribute this difference to the fact that they do not correct for different user costs between the two factors and use data from Pittsburgh, declining city where land prices are perhaps expected to decline. The fact that land values in their data is assessed may also play a role. When we implement their approach on our data XXX...

2. Housing: treating output as a latent variable

House builders competitively produce housing services using land T and non-land inputs K, which we refer to as capital for convenience. They face a production technology $H(K,T)$ assumed to be strictly increasing and concave in both arguments. Land is exogenously partitioned into parcels of area T where T is distributed over $[\underline{T},\overline{T}]$. At a given location x, each unit of housing can fetch a price $P(x)$. This price is taken as given by builders. For a parcel of area T located at x, the builder's profit is

$$\pi(x) = P(x)H(K,T) - rK - R(P(x),T)\,, \tag{1}$$

where r is the user cost of capital (i.e., non-land) inputs and $R(P(x),T)$ is the (rental) price of a parcel of land of area T at location x.

The first-order condition for profit maximisation with respect to capital inputs is

$$P(x)\frac{\partial H(K,T)}{\partial K} = r\,. \tag{2}$$

The optimal amount of capital inputs that satisfies this condition is given implicitly by $K^* = K^*(P(x),T)$. As the housing production function is strictly increasing in capital, there is a bijection between the price of housing and the profit-maximising level of capital, holding the area of the parcel fixed: $P(x) = P(K^*,T)$. Hence, we can rewrite the price of land $R(P(K^*,T),T)$ directly as a function of the profit-maximising level of capital inputs K^* instead of the price of housing: $R(P(K^*,T),T) \equiv R(K^*,T)$.

Free entry implies that all the rents from building are dissipated into the price of land so that

$$R(K^*,T) = P(K^*,T)H(K^*,T) - rK^*\,. \tag{3}$$

We note that the price of land is uniquely defined for any K^* and T. While the price of land is observed in the data, the price of housing is not. We can insert equation (2) into (3) to eliminate the price of housing, $P(x) = P(K^*,T)$. We obtain the following partial differential equation:

$$\frac{\partial H(K^*,T)}{\partial K^*} = \frac{r}{rK^* + R(K^*,T)}H(K^*,T)\,. \tag{4}$$

For consistency with our empirical work below, this expression may be more intuitively rewritten as an elasticity:

$$\frac{\partial \log H(K^*,T)}{\partial \log K^*} = \frac{rK^*}{rK^* + R(K^*,T)}\,, \tag{5}$$

where log denotes a natural logarithm. Because $K^* = K^*(P(x),T)$, both sides of equation (4) depend on parcel location x. It is also important to keep in mind for later that, although location intervenes in both the first-order condition (2) and in the zero-profit condition (3), it only does so through its effects on the price of housing, which, in turn, affects the price of land and the level of capital inputs. In particular location does not affect the production of housing directly in $H(K,T)$.

Consider that for a given parcel of area T, the desirability of locations varies such that the price of housing is distributed over the interval $\left[\underline{P},\overline{P}\right]$. The optimal level of capital investment in housing K^* then covers the interval $\left[\underline{K},\overline{K}\right]$ where $\underline{K} = K^*(\underline{P},T)$ and $\overline{K} = K^*(\overline{P},T)$. The solution to the differential equation (5) for a given value of the optimal amount of capital inputs K^* in this interval is obtained by integration and can be written as:

$$\log H(K^*,T) = \int_{\underline{K}}^{K^*} \frac{rK}{rK + R(K,T)} \mathrm{d} \log K + \log Z\left(T\right) . \tag{6}$$

where $Z(T)$ is a positive function. Equation (6) enables the computation of the number of units of housing for a property on a parcel of area T, knowing the price of that parcel and the amount of capital invested to build this property. In addition, the computation uses the profit-maximising choices made for other parcels of the same area T for which less capital was invested. In turn, the reason why these parcels received less capital is that they were located in less desirable locations where the demand for housing is less.

To illustrate the workings of equation (6)and check the consistency of our approach, consider first a Cobb-Douglas situation. In this case, the price of land R and the price of capital invested to build the property $r\,K$ are proportional. This implies that the term within the integral is constant.[8] As a result, $\log H$ is proportional to $\log K$ and housing services are proportional to the investment made to build the property elevated to some power. Put differently, we retrieve a Cobb-Douglas form.

To take another example, assume now that the production function enjoys a constant elasticity of substitution between land and capital equal to two. In this case, profit maximisation implies that capital inputs should increase with the square of the price of parcel of area T. Integrating the share of capital as in equation 6 implies that the production of housing is proportional to $(\sqrt{K}+b)^2$ where b is a constant. This functional form is indeed the generic functional form for a CES production function when, with an elasticity of substitution equal to two, and a factor, land, is held constant.

[8]This property was already noted by Klein (1953) and Solow (1957).

An important assumption of our model is that the price of land for a parcel is affected by its location x only through the price that residents are willing to pay to live at location x on the demand side. Our approach so far does not allow for a parcel characteristic y to affect the production technology directly. To understand the implications of potential supply differences, let us consider first a simple example where all parcels are of unit size, the demand for housing is the same at all locations: $P(x) = P = 1$, and the price of capital inputs is normalised to unity: $r = 1$. Assume that housing is produced following $H(K,y) = \frac{1}{a}K^a\, y^{1-a}$ where the the unobserved characteristic y measures the ease of construction. Then, in equilibrium capital is given by $K(y) = y$ and parcel values capitalise the ease of construction, $R(y) = \frac{1-a}{a}y = \frac{1-a}{a}K$. Using equation (6) to estimate the value of housing, we would obtain that the production of housing is proportional to K instead of being proportional to K^a.[9]

More generally, assume that parcels are now characterised by two location characteristics, x and y. The characteristic x still affects the price that residents are willing to pay $P(x)$ while y affects the production of housing directly. The production of housing is now given by $H(K,T,y)$. The analog to the first-order condition (2) is now $P(x)\partial H(K(x),T,y)/\partial K = r$. The zero-profit condition also implies that y affects the price of land directly: $R(K^*,T,y) = P(x)H(K^*,T,y) - rK^*$. The partial differential equation analogous to (4) is now:

$$\frac{\partial H(K^*,T,y)}{\partial K^*} = \frac{r}{rK^* + R(K^*,T,y)} H(K^*,T,y)\,. \tag{7}$$

It can be solved only for a given y. Integrating as we do in equation (6) ignoring y will be problematic since y will be correlated with both the quantity of housing H and the price of land R. Locations with a particularly good y will both be able to generate more housing given K and face a higher price for land. Below, we develop an instrumental-variable approach to solve this problem.

Before going forward, some of our other assumptions must be discussed further. First, we assume non-increasing returns to capital. This is arguably an appropriate assumption empirically for newly constructed single-family homes. Second, because of the ease of entry in this industry and the absence of fine product differentiation, our assumption of competitive builders also strikes us as

[9] Note that this problem of missing variable is worse than in standard cases because it creates a bias even when the missing characteristic y is uncorrelated with P.

reasonable.[10] Third, at every location, the unit price of housing is taken as given by competitive builders. We thus implicitly assume an integrated housing market. This is defensible in our empirical application below where we can ignore outliers and where we consider the construction cost of 'undecorated' houses, that is before they become fully customized.[11] Finally, parcels are exogenously determined. Treating land as a fixed input is reasonable in France where zoning rules usually prevent the subdivision of existing parcels in residential areas. Potential parcel selection issues are also discussed below with our empirical strategy.

3. Estimation of the housing production function

3.1 Taking our framework to the data

For parcels of a given area T, equation (6) can be used to estimate housing production as a function of capital inputs up to an integration constant. Our (partial) identification relies on the fact that housing prices differ across locations. When holding land area T constant, spatial variations in housing prices lead to spatial variations in land prices and in profit-maximizing capital inputs. We can then use variations in land prices (which we observe) instead of variations in housing prices (which we do not observe). By equation (6), estimating the production of housing at a given level of capital inputs K^* requires information about the price of the parcel $R(K,T)$ for all the values $K \in [\underline{K},K^*]$.

Our approach imposes two requirements on the data: K^* must be continuous and there must be a bijection between R and K^* given T. In the data, the price of parcels of a given area T is observed only for some values of capital, not for the entire continuum. For any pair (K,T), we can use the fact that we will have observations with slightly larger and slightly lower values of K for a given T as well as observations with slightly lower and slightly larger values of T for a given K. That is, we can estimate the price of land for every value of K and T using a kernel non-parametric regression. The kernel we use is the product of two independent normals and the bandwidth is computed using

[10] A search on the French yellow pages (`http://www.pagesjaunes.fr/`) yields 1783 single-family house builders for Paris (largest urban area with population above 12 million), 111 for Rennes (10th largest urban area with population 654,000), and still 38 for Troyes (50th largest urban area with population 188,000). (Search conducted on 21st May 2013 looking for 'constructeurs de maisons individuelles' – builders of single family homes – typing 'Ile-de-France' to capture the urban area of Paris, 'Rennes et son agglomération', and 'Troyes et son agglomération' for the other two cities.)

[11] Contracts between buyers and builders are usually for undecorated houses. After this step is completed, buyer can choose more specific items such as options for their kitchen or bathrooms or their flooring to fully customise their property. About 70% of our observations are for undecorated houses. For the other observations we condition out their decorated status.

a standard rule of thumb for the bivariate case (see Silverman, 1986).[12] The estimated price of land is given by the following formula:

$$\widehat{R}\left(K,T\right)=\sum_{i}\omega_{i}R\left(K_{i},T_{i}\right)\ \text{ with }\omega_{i}=\frac{L_{h_{K}}\left(K-K_{i}\right)L_{h_{T}}\left(T-T_{i}\right)}{\sum_{i}L_{h_{K}}\left(K-K_{i}\right)L_{h_{T}}\left(T-T_{i}\right)} \tag{8}$$

where N is the number of observations, $L_{h}\left(x\right)=\frac{1}{h}f\left(\frac{x}{h}\right)$ with $f\left(\cdot\right)$ the density of the normal distribution, and $h_{X}=N^{-1/6}\sigma\left(X\right)$ with $\sigma\left(X\right)$ the empirical standard deviation for variable X computed from the data. This kernel estimator has the property of making $R(K,T)$ unique, which is requested by equation (3).

Note also that equation (6) involves the computation of an integral for which the lower bound is the lowest value of the profit-maximising capital. In practice, we can potentially use any value of capital as lower bound, $\underline{K}$. Empirically, there is a trade-off. A small value for the lower bound will allow us to study the variations of the housing production function over a wide range of values for capital inputs but this may come at the price of being in a region where there are few observations. In our work below, we work restrict attention to observations above the first decile (and below the ninth decile) to estimate the production of housing.

Once the production of housing is at hand, we regress it on capital inputs. For instance, under Cobb-Douglas, for any fixed T, there should be a linear relationship between the logarithm of the housing production function and the logarithm of capital. In other words, we use a non-parametric estimate of housing to infer how housing parametrically varies with capital as our ultimate goal is to be able to characterise the production function for housing. This said, to make pronouncements about the nature and shape of the production function of housing, we face the standard problem that there is no undisputable 'best' specification when regressing the (estimated) quantity of housing on capital investment. One might be tempted to reject a given specification because terms of higher order are significant. For instance, we can reject the Cobb-Douglas specification because log capital squared is significant. The pitfalls of doing this are well-known. No specification might be fully robust to the inclusion of further terms (or there might be several). An alternative is to assess the goodness of fit of our specifications using an information criterion that tradeoffs a measure of goodness of fit against the number of explanatory variables. We view this type of approach as being of limited value since the penalty imposed for the number of explanatory variable is arbitrary. We also note that the R-squared of the parametric regressions reported below is always extremely high

[12] Alternatively, we could consider that the integrand $\frac{rK}{rK+R(K,T)}$ is computable only at the observed values of capital and recover that integrand for other value of capital using a kernel non-parametric regression of the integrand.

because we use data that are smoothed in the non-parametric estimation. Instead, we rely on the magnitude of the prediction differences implied by different specifications. To put it simply, we claim that the Cobb-Douglas form provides a good approximation. Even though we can formally reject this functional for, because higher-order terms are significant, the differences in prediction that these added terms make is minimal.

3.2 Identification and estimation strategy

As mentioned above, the main worry with our approach is the following. As made clear by the zero profit condition (3), we rely on the fact that the price of land for a parcel only reflects the price that housing can fetch on this parcel. It is easy to imagine situations where the price of a parcel also reflects the cost of construction. A parcel may, in equilibrium, be worth less because its terrain has a steep slope and building on it would be more costly.

To correct for this potential problem, we develop an instrumental variable approach. Conceptually, we want to estimate the following regression:

$$\log R_i = X_i\, a^R + Y_i\, b^R + \epsilon_i^R\,, \tag{9}$$

where X is a vector of location characteristics which affect the demand for housing and Y is a vector of location characteristics that affect the supply of housing. The vector X is the empirical counterpart of the location effect x in our framework above while Y is the empirical counterpart to y. Then we can build a predicted land price $\widehat{R}(K,T)$ which depends only on demand characteristics and not on supply characteristics: $\log \widehat{R}(K,T) = X\, a^R + \overline{Y}\, b^R$ where $\overline{Y}\, b^R$ is the mean effect of supply characteristics. Note that we do not want to use any information about the error term which is also likely to contain unobserved supply characteristics. We can then use predicted parcel prices instead of the actual parcel prices in the empirical counterpart of equation (6).

The location characteristics Y that affect the price of land through the supply of housing will also affect capital inputs K. Hence we also want to estimate the following regression analogous to equation (9) for capital inputs:

$$\log K_i = X_i\, a^K + Y_i\, b^K + \epsilon_i^K\,. \tag{10}$$

Like with land prices, we can then compute a predicted value for capital inputs $\widehat{K}$ which depends only on demand characteristics: $\log \widehat{K} = X\, a^K + \overline{Y}\, b^K$ where $\overline{Y}\, b^K$ is the mean effect of supply characteristics. We can then use predicted investments instead of actual investments in the empirical counterpart of equation (6).

Finally, recall that our model takes land area T as given. It may be that the location characteristics that affect supply also affect parcel area. For instance, steeper terrains that make building more difficult may be divided into bigger parcels. Hence, land area might be correlated with Y and this may also affect our results. In our example, larger parcel will look more capital intensive because of their steeper terrain. This suggests applying the same approach as in equation (9) to land area and estimate:

$$\log T_i = X_i\, a^T + Y_i\, b^T + \epsilon_i^T\, . \tag{11}$$

The resulting predicted value of land $\widehat{T} = X\, a^T + \overline{Y}\, b^T$ can then be used in our estimation of equation (6) instead of the actual land area.

In practice, we can estimate regressions of the following type:

$$\log R_i = \beta_{c(i)} + \delta_{c(i)}\, d_i + B_i\, \gamma + \epsilon_i\, , \tag{12}$$

where $\beta_{c(i)}$ is an effect of the urban area c where parcel i is located, d_i is a measure of distance of parcel i to the centre of its urban area, and B_i is a vector of other characteristics for parcel i such as how this parcel was purchased, whether it is serviced, etc. Analogous regressions where the dependent variable is K and T can also be estimated. Note that the estimation of housing production H requires information about land prices, capital investment, and land area. For each of these three quantities we can use either the actual value or the predicted value. To avoid too many permutations, we report results for the actual values (R, K, T) and for the predicted values ($\hat{R}$, $\hat{K}$, $\hat{T}$). We experimented with mixes of actual and predicted values such as ($\hat{R}$, $\hat{K}$, T). They implied results very similar to those reported here.

The key problem is that it is not obvious whether any explanatory variable we might use belongs to the demand determinants X or the supply determinants Y (including any mix of the two). From theory, we expect the city effect, β_c, and the distance to the city centre, δ_c, to be strong determinants of the demand for housing in line with monocentric urban models in the tradition of Alonso (1964), Muth (1969), and Mills (1967). In the simplest version of these models, the price of housing, land prices, capital inputs used in housing, and parcel area at each location are actually fully explained by the distance to the centre and the price of housing at this centre (Duranton and Puga, 2015). At one extreme, a minimalist approach would be to consider only city effects and distance to the center in X. One may choose to consider richer specifications for X including higher order distance terms or other characteristics of parcels. While richer specifications lead to better predictors of $R(K,T)$, which is important for an accurate estimation of housing quantities in equation (6), they may also lead us to include explanatory variables that should be part of Y and conditioned out.

We do not take any strong stance regarding which approach is preferable for the specification of equation (9) and experiment with both simple and rich specifications for the vector of demand determinants X.

We note that our identification strategy relies on the same principle as standard instrumental variables approaches in that we rely on the variation of the variables of interest that come from surrogate variables (urban area of belonging and distance to its centre) instead of their full variation. While the main principle is the same, our implementation differs considerably from standard two-stage least-squares procedures. Our objective is to provide a non-parametric estimate of housing, H, as a function of capital, K, and the associated land value, R, for a given land area, T, before regressing $\log H$ on $\log K$. Given that unobserved supply characteristics of parcels are expected to affect capital investments, land values, and land areas, our 'first stage' (or, more appropriately, first step) generates predicted values for three variables, capital $\hat{K}$, land values $\hat{R}$, and land area $\hat{T}$. We can then estimate a predicted value of housing $\hat{H}$. Finally, we can regress $\hat{H}$ on $\hat{K}$ to characterise (or parameterise) the production function of housing.

Although we may use only one instrument for three variables in the first step, the effect of capital inputs on housing is nonetheless identified. This is because we associate a unique value of land prices for each level of investment for a given land area. Hence, we should be thinking as our surrogate variables instrumenting a triplet (K,R,T) (or only part of that triplet) and not three different explanatory variables.[13] Then note also that our second step estimates an instrumented value of housing before regressing it on instrumented capital in a third step. To be able to parameterise the structural relationship between housing and capital for a given land area, we need to use instrumented values for both the dependent and the explanatory variable.

In addition to our main identification issue, one may worry about various forms of supply heterogeneity across cities. In particular, construction costs to vary across cities (Gyourko and Saiz, 2006) and land prices capitalise (expected) city growth (Combes, Duranton, and Gobillon,

[13] In effect we use predicted values of K and R to estimate H given a predicted T. We then regress the estimated value of H on predicted K, again for a predicted T.

2013). The solution would be to estimate a separate housing production function for each urban area separately. Except for the very largest French cities, the number of land transactions in a typical urban area is unfortunately too small to do this. We can nonetheless divide urban areas in size classes and perform a separate estimation for each size class.

Measurement error on capital inputs K or land prices R may also affect our estimation of housing quantities. Measurement error is dealt with in two different ways in our approach. First, as mentioned above, we kernel-smooth the land price data. Second, our IV approach will use predicted land prices which are less prone to measurement error than observed land prices. Just like standard instrumental variable approaches, our IV approach will allow our estimation to (i) focus on sources of land prices variation that are driven solely by the demand for land and (ii) reduce measurement error.

Finally, we need to keep in mind that housing construction is tightly regulated in France as in many other countries. The two main regulatory instruments are (i) the zoning designation and (ii) the maximum intensity of development.[14] The zoning designation indicates whether a parcel can be developed and, if yes, whether it can be done so for residential purpose. Given that we only observe parcels with a development permit for a single-family home, this creates no further issue beyond the fact that we estimate the production function for single-family homes in parcels designated for that purpose. Turning to the maximum intensity of development that applies to a parcel, it is not known in the data. Although we note that the quantity of housing is not solely determined by the square-footage of a house, we nonetheless acknowledge that we estimate the production function for housing under prevailing restrictions on residential development. Absent regulations, single family homes may be extremely different from what they are under the current regulatory regime.[15] However, we are interested in estimating how land and capital inputs can be transformed into housing under the current regulatory constraints. While knowing how regulations affect the production of housing on parcels that can be developed is certainly a question of interest, this is not one that we can answer here.

[14]The maximum intensity of development is essentially a standard floor-to-area (FAR) regulation. In France, it is referred to as the 'coefficient d'occupation des sols' (COS). This regulation is subject to national guidelines but can be adjusted locally by the municipality. Other regulations such as minimum parcel size or an obligation to follow a local style with more or less stringency also often apply.

[15]To be concrete, consider that two technologies with very different production functions are available to build housing. If one technology is banned through regulations, we can only learn about the second.

3.3 Implementation

To implement our approach, we first consider a fine grid of 900 by 900 values of capital inputs and land values which we obtain by kernel-smoothing our observations as described above. We then consider each of the nine deciles of land area. For decile of land area, we set the lower bound of integration $\underline{K}$ in equation (6) to its bottom decile and the highest value we consider $\overline{K}$ is the top decile. This avoids having our estimations influenced by potential outliers or a different market segment. Finally, the housing production function is estimated for 900 values of the capital which are uniformly distributed in the interval defined by the first and last decile of capital. The computations involve the estimation of the price of land at any point of the grid using equation (8) and the evaluation of the second-right hand-side term in equation (6) using trapezoids to approximate the integral in that term.

Before turning to our results, several implementation issues must be discussed. First, our model relies on the rental value of land and the rental value of capital inputs. The data we use only report the price of land and the cost of construction. Using stocks (transaction values) instead of flows (rental values) makes no difference to our approach when the user cost of land is the same as the user cost of capital. This is not the case when the user costs differ across factors.[16] Following Combes *et al.* (2013) we take the annual user cost of capital to be 6%. This reflects a long term interest rate of 5% and a 1% annual depreciation.[17] For the user cost of land, we take a value of 3% per year.[18]

Second, structure values and land prices are influenced by inflation over the period. To make quantities comparable across time, we correct them for year effects. For that purpose, we regress the logarithm of each quantity on year fixed effects for each year of data. Net quantities are obtained by subtracting year fixed effects from their logarithm and taking the exponential.

Third, confidence intervals are estimated by bootstrap. At each iteration, a random sample is drawn with replacement from the universe of all transacted land parcels. The price of land is recomputed at each point of our grid of values for the area of land and capital using kernel

[16] This is very much in the spirit of the user-cost correction first proposed by Poterba (1984).

[17] According to Commissariat Général au Développement Durable (2012), the difference between investment in housing and the increase in housing stocks in the French national account is 2009 was about 15 bn Euros, which corresponds to slightly less than 1% of GDP.

[18] As estimated by Combes *et al.* (2013), the elasticity of land prices to local income is slightly above 1 while the elasticity of the price of land with respect to population is slightly below one. A 1% annual increase in income and a 1% annual increase in population (the mean urban population growth in France in the recent past) thus imply an about 2% annual appreciation of land prices to be deducted from the long run interest rate of 5%.

non-parametric regressions, and the housing production is reevaluated. The distribution of values for housing production at each point of the grid is then recovered and confidence intervals can be deduced.

4. Data

The observations in our data are transacted land parcels that are extracted from the French Survey of Developable Land Price (*Enquête prix des terrains à bâtir*, *EPTB*). This survey is conducted every year in France since 2006 by the French Ministry of Ecology, Sustainable Development, and Energy. The sample frame is drawn from Sitadel, the official land registry, which covers the universe of all building permits for detached houses. The survey selects single-family homes owned by households. Permits for extensions to existing houses are excluded. Our study period is from 2006 to 2012.

Parcels are drawn randomly from all municipal strata (about 3,700 of them). Each stratum corresponds to a group of municipalities (about 36,000 in France). Two thirds of the permits are surveyed. Some French regions paid for a larger sample in some years. In total, this corresponds to about 120,000 permits per year on average during our study period. The survey is mandatory and the response rate after one follow-up is above 75%. Annually, the number of observations ranges from 48,991 in 2009 to 127,479 in 2012.

While it is possible to get houses built in many ways in France, the arrangement we study covers a large fraction of new constructions for single-family homes.[19] Getting a new house by first buying the land and then getting a house built on it appears fiscally advantageous as it avoids paying stamp duties on the structure. This arrangement also greatly reduces financing constraints for house builders and lowers their risks.[20]

For each transaction, we know the price of the parcel, its municipality, how it was acquired (purchase, donation, inheritance, other), some information about its buyer, whether the parcel was acquired through an intermediary (a broker, a builder, another type of intermediary, or none), and some information about the house built, including its cost. The notion of building costs may be ambiguous but we know whether the reported cost reflects the cost of a fully decorated house or

[19] The consultancy Développement-Construction reports between 120,000 and 160,000 new single family homes in total during the period (`http://www.developpement-construction.com/`.

[20] We do not observe large groups of observationally similar houses built at the same location in the same year. When such houses are purchased together with the land, they are outside the coverage of the survey we use. While the development of large subdivisions with many standardized houses is frequent in many parts of the US and were once important in France, it is less frequent today as French planning laws strongly favour in-filling at the expense of large new developments at the urban fringe.

the cost of a house prior to full completion (i.e., excluding interior paints, light fixtures, faucets, kitchen cabinets, etc). Undecorated houses represent the large majority of our observations. We also know the area of a parcel and whether it is 'serviced' (i.e., has access to water, sewerage, and electricity). We only retained parcels that were purchased and ignore inheritances and donations.

Table 1: Descriptive Statistics

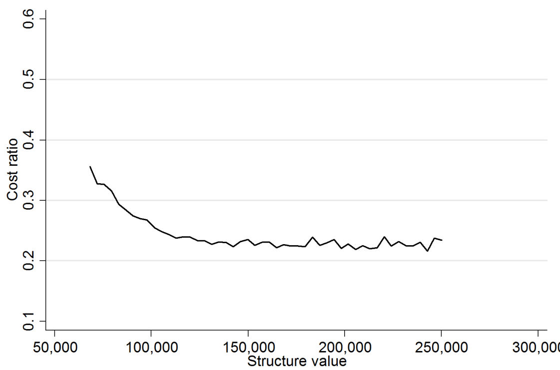
Variable	Mean	St. deviation	1st decile	Median	9th decile
Entire country:					
Parcel area	1,156	947	477	883	2,079
Construction cost	127,551	55,003	78,440	115,000	190,667
Parcel value (2012 Euros)	63,387	58,164	19,673	50,000	120,000
Parcel value per m^2 (2012 Euros)	80	86	14	58	166
Urban areas:					
Parcel area	1,048	821	449	820	1,883
Construction cost	131,616	57,599	80,140	118,000	199,750
Parcel value (2012 Euros)	73,115	62,518	27,017	58,271	135,000
Parcel value per m^2 (2012 Euros)	96	94	22	72	192
Greater Paris:					
Parcel area	839	673	329	665	1,493
Construction cost	151,298	73,727	89,173	132,850	236,605
Parcel value (2012 Euros)	142,010	108,598	69,155	124,419	220,000
Parcel value per m^2 (2012 Euros)	237	193	67	182	466

Notes: The sample contains 386,181 observations. Quantities are deflated using the consumption price index.

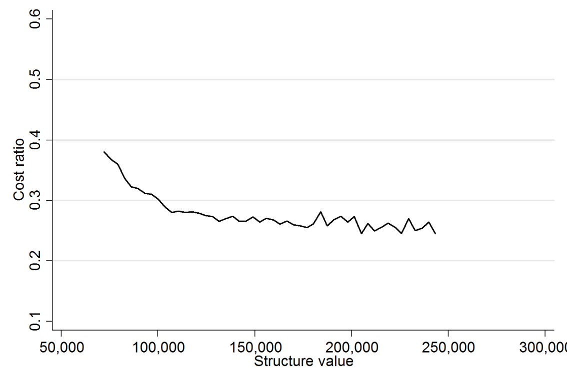
Table 1 provides descriptive statistics for all our main variables. The first interesting fact that emerges from table 1 is the considerable variation in parcel area, total construction costs, and parcel value per square meter. A parcel at the top decile is about four times as large as a parcel at the first decile. Interestingly, for construction costs, the corresponding inter-decile ratio is only about 2.4 whereas for unit land prices, it is nearly 12. The second interesting feature of the data that table 1 highlights is that this variation does not only reflects a rural vs. urban gap. Even when we consider only transactions from Greater Paris we still observe considerable variation in parcel values per square meter.

Before presenting our main results, figure 1 proposes four plots of the ratio of the value of land to the total cost of the house after deflating each factor by its user cost. Panel (a) represents this plot for the entire country; panel (b) restricts its observations to urban areas, panel (c) restricts its observations even further to urban areas with population between 50,000 and 100,000 whereas panel

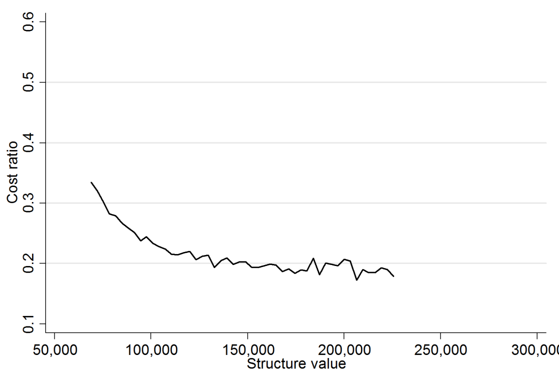
Figure 1: Land to structure cost ratios



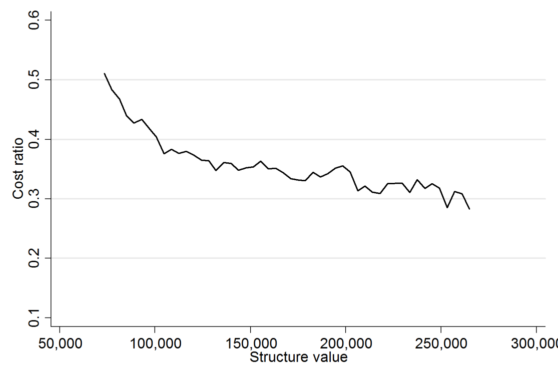
Panel (a) Entire country



Panel (b) All urban areas



Panel (c) Urban areas, 50,000-100,000

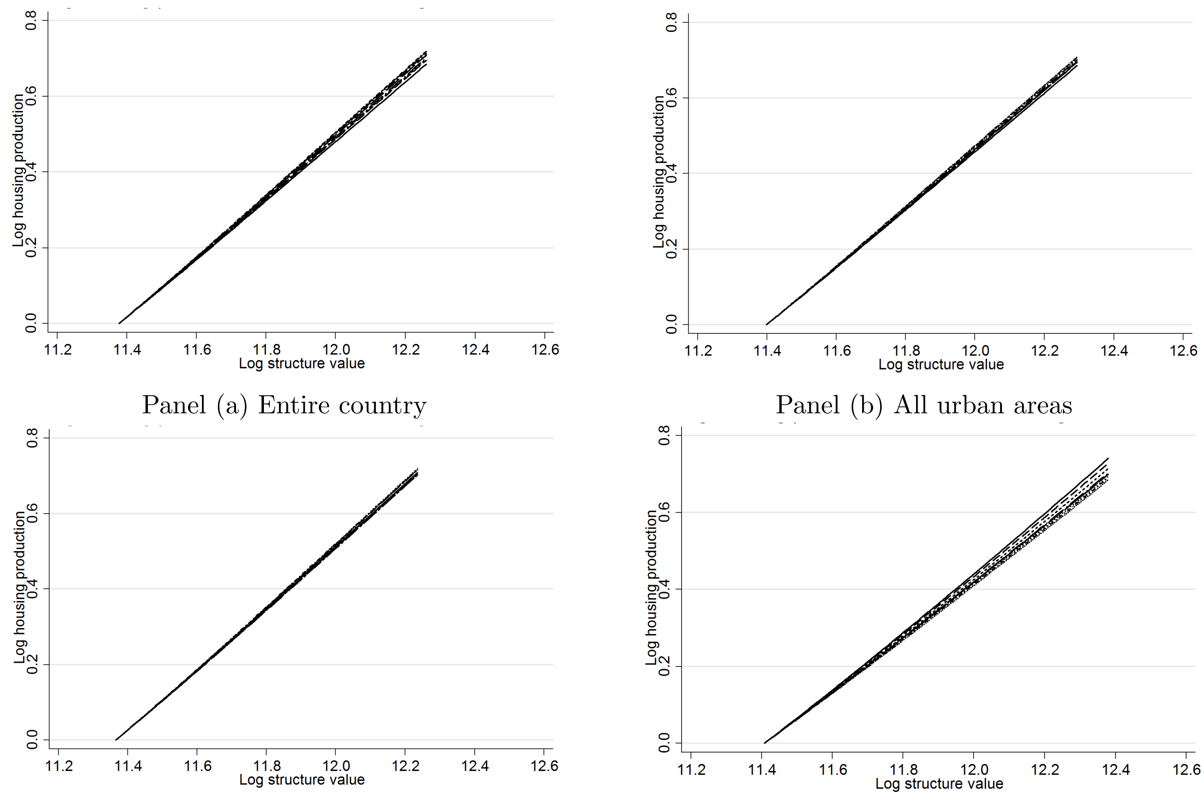


Panel (d) Urban areas, more than 500,000 (excl. Paris)

Notes: Observations between the 5th and 95th percentile of structure values in each panel.

(d) only considers the largest cities (bar Paris) with population above 500,000. A Cobb-Douglas production function predicts that the cost ratios plotted in these four figures should be flat and constant across figures. While for at least the first three panels, the cost ratio seems constant after a construction cost of 100,000 Euro, it is decreasing in construction costs for values below this threshold. We can also observe that the cost ratio of land is also higher in large urban areas where it usually stands above 30% whereas it is about 20% in small urban areas. This said, it is unclear how we should interpret these plots. Land parcels on which construction is particularly cheap will fetch a higher price and the ease of construction will also affect the investment in capital. These plots also arguably suffer from a serious measurement problem. We expect constructions costs to be reported with error. This affects both the value reported on the horizontal axis and the cost ratio reported on the vertical axis. As a result, it is difficult to make definitive conclusions about the production function for housing from the plots reported here since we expect the measured relationship between construction costs and the cost ratio to be downward biased.

Figure 2: log housing production as a function of log capital investment



Panel (a) Entire country

Panel (b) All urban areas

Panel (c) Urban areas, 50,000-100,000

Panel (d) Urban areas, more than 500,000 (excl. Paris)

5. Results

5.1 Main results using the raw data

Before looking at formal estimation results, it is useful to visualise our non-parametric estimations. Each panel of figure 2 plots the estimated log production of housing, $\log H$, as a function of capital investment, $\log K$ for every decile of land area T. This is the empirical equivalent to equation (6) in our framework. Panel (a) of figure 2 represent the production function for housing for the entire country while panels (b), (c), and (d) do the same for all urban areas, small urban areas with population between 50,000 and 100,000 and large urban areas with population above 500,000 (excluding Paris), respectively.

Although we must remain cautious when visualising these results, several remarkable features emerge from figure 2. First, as might be expected, housing production always increases with capital. More specifically, log housing as a function of log capital investment is apparently linear with a slope of about 0.80. This is of course consistent with a Cobb-Douglas production function with an

Table 2: Housing quantities and capital inputs, OLS by area decile

Decile	1	2	3	4	5	6	7	8	9
Panel (a)									
$\log(K)$	0.779^a	0.792^a	0.796^a	0.800^a	0.808^a	0.814^a	0.819^a	0.820^a	0.818^a
	(0.00015)	(0.00018)	(0.00020)	(0.00020)	(0.00020)	(0.00017)	(0.00018)	(0.00018)	(0.00018)
R^2	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Observations	900	900	900	900	900	900	900	900	900
Panel (b)									
$\log(K)$	0.360^a	0.267^a	0.228^a	0.257^a	0.266^a	0.399^a	0.362^a	0.361^a	0.329^a
	(0.00566)	(0.00691)	(0.00857)	(0.00981)	(0.01014)	(0.01126)	(0.01083)	(0.00984)	(0.01004)
$[\log(K)]^2$	0.018^a	0.022^a	0.024^a	0.023^a	0.023^a	0.018^a	0.019^a	0.019^a	0.021^a
	(0.00024)	(0.00029)	(0.00036)	(0.00041)	(0.00043)	(0.00048)	(0.00046)	(0.00042)	(0.00042)
R^2	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Observations	900	900	900	900	900	900	900	900	900

Notes: OLS regressions with a constant in all columns. XXXXRobust standard errors in parentheses. a, b, c: significant at 1%, 5%, 10%.

exponent of about 0.80 for capital. Second, the relationship between $\log H$ and $\log K$ seem very similar for all deciles of parcel land area. Although we can identify the production function of housing only for each quantile of land area, the minimal differences that appear across deciles in figure 2 are suggestive of a similar technological relationship between land and capital on small and big parcels alike. The last important feature of figure 2 regards the differences across panels. While the relationship $\log H$ and $\log K$ is very much the same across the first three panels, the last panel is modestly different with more dispersion across deciles and a slightly lower slope.

We now explore our preliminary results more formally using regressions. Our first set of results is reported in panel (a) of table 2 where, for each decile of land area, we regress the estimated log production of housing on log capital investment in structure. Each regression relies on 900 observations obtained after smoothing the original data as per formula (8) and deleting the bottom and top 10%. Each column of table 2 corresponds to a separate decile. Hence, the estimated share of capital for the first decile is 0.78. It is 0.79 for the second decile, 0.80 for the third and fourth, then 0.81 for the fifth and sixth, and finally 0.82 for the last three deciles. While these shares are not exactly constant across deciles, the differences remain small. Interestingly, the share of capital is estimated to be larger in larger parcels. This would be consistent with land and capital

Table 3: Housing quantities and capital inputs, OLS by class of city size

City size class	Country	Urban areas	0-50	50-100	100-200	200-500	500+	Paris
Panel a								
$\log(K)$	0.805^a	0.784^a	0.832^a	0.822^a	0.814^a	0.785^a	0.730^a	0.700^a
	(0.00016)	(0.00011)	(0.00012)	(0.00012)	(0.00010)	(0.00010)	(0.00023)	(0.00015)
R^2	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Observations	8,100	8,100	8,100	8,100	8,100	8,100	8,100	8,100
Panel b								
$\log(K)$	0.315^a	0.365^a	-0.075^a	0.038^a	0.230^a	0.068^a	-0.091^a	-0.002
	(0.01577)	(0.00978)	(0.00752)	(0.00920)	(0.00837)	(0.00647)	(0.01929)	(0.01213)
$[\log(K)]^2$	0.021^a	0.018^a	0.038^a	0.033^a	0.025^a	0.030^a	0.034^a	0.029^a
	(0.00067)	(0.00041)	(0.00032)	(0.00039)	(0.00035)	(0.00027)	(0.00081)	(0.00051)
R^2	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Observations	8,100	8,100	8,100	8,100	8,100	8,100	8,100	8,100

Notes: OLS regressions with decile fixed effects in all columns. XXXXRobust standard errors in parentheses. *a*, *b*, *c*: significant at 1%, 5%, 10%.

being (weakly) complement, that is with an elasticity of substitution between land and capital just below one. Because these estimates are subject to a number of identification worries, we refrain from further conclusions for now but note that in any case the differences in the production function across quantiles are economically small. Importantly, we also note that our linear regressions provide a near perfect fit as the R^2 is always above 0.999.

[[Paragraph about corrected standard errors]]

Panel (b) of table 2 replicates the regressions of panel (a) adding the square of log housing capital as an explanatory variable. We note that the quadratic term is significant in all regressions with a coefficient of about 0.02. Hence, the production function for housing is not strictly linear but convex. Because log capital typically varies between about 11.3 at the bottom decile and, 12.2 at the top decile, this convexity implies that the share of capital is only about 0.02 larger for houses built at the top decile of capital relative to houses built at the bottom decile. While the housing production function is convex, this convexity is minimal and the differences in the share of capital between the largest and smallest houses are tiny. We also note that this feature is consistent with the cross-decile differences from the previous panel and the existence of small complementarities between land and capital.

Panel (a) of table 3 regresses again the log of estimated housing capital on log capital investment but this time considers different samples corresponding to different geographies. In each regression, all the land area deciles are lumped together but decile fixed effects are included. The first column considers the entire population of transactions. The estimated coefficient on capital investment is 0.80 which is about the mean of all deciles in panel (a) of table 2. Column 2 considers only observation from urban areas and estimates a marginally lower coefficient of 0.78. The following 6 columns consider urban areas of increasing sizes. For the smallest urban areas with population below 50,000 the estimated share of capital investment is 0.83. This coefficient is 0.73 for large urban areas with population above 500,000 and 0.70 for Paris. Because land is more expensive in larger cities, these results are again consistent with a weak complementarity between land and capital in the production of housing.

Panel (b) of table 3 repeats the same exercise as panel (a) adding a quadratic term for the log of capital investment. Just like panel (b) of table 2 it provides evidence of a modest convexity.

5.2 Instrumental variable results

We now turn to our results when we allow for R, K, and T to be simultaneously determined. We first estimate equations (9), (10), and (11) using the urban area a parcel belongs to and, for each urban area, the distance to its center as sources of exogenous variation (i.e., as instruments) to determine R, K, and T. These regressions also include a number of controls for parcel characteristics. As argued above, the urban area where a parcel is located and the distance to its center are plausibly exogenous determinants of its area, price, and capital investment. To proxy for some potentially important unobservables we also include the degree of completion, the intermediary through which the parcel was purchased, and a dummy for access to services as controls. As noted in Combes *et al.* (2013), urban area fixed effect and (log) distance to the centre explain 57% of the variation of the price of land per square meter in the data for 2006. Although we do not develop a procedure to test for weak instruments in our context, there is no doubt that the two variables we use strongly predict our potentially endogenous variables.

The specification of table 4 mirror exactly those of table 2. For each instrumented decile of land area, panel (a) reports results of a linear regression of instrumented $\log H$ on instrumented $\log K$. The results are very similar to the un-instrumented results of table 2. The fit of these linear

Table 4: Housing quantities and capital inputs with endogenous R, K, and T, OLS by area decile

Decile	1	2	3	4	5	6	7	8	9
Panel (a)									
$\log(K)$	0.787^a	0.784^a	0.784^a	0.774^a	0.784^a	0.795^a	0.806^a	0.813^a	0.814^a
	(0.00036)	(0.00038)	(0.00046)	(0.00065)	(0.00047)	(0.00038)	(0.00033)	(0.00016)	(0.00054)
R^2	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Observations	900	900	900	900	900	900	900	900	900
Panel (b)									
$\log(K)$	2.512^a	3.242^a	4.152^a	5.694^a	4.317^a	3.069^a	2.233^a	1.215^a	4.477^a
	(0.074)	(0.055)	(0.043)	(0.043)	(0.031)	(0.063)	(0.073)	(0.038)	(0.071)
$[\log(K)]^2$	-0.073^a	-0.104^a	-0.142^a	-0.208^a	-0.149^a	-0.096^a	-0.060^a	-0.017^a	-0.155^a
	(0.00315)	(0.00234)	(0.00184)	(0.00185)	(0.00131)	(0.0027)	(0.0031)	(0.00164)	(0.00303)
R^2	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Observations	900	900	900	900	900	900	900	900	900

Notes: OLS regressions with a constant in all columns. XXXXRobust standard errors in parentheses. a, b, c: significant at 1%, 5%, 10%.

specifications is high and the estimated share of capital is about 80% on average across deciles. The share of capital also modestly increases across deciles of land area. If anything, this increases is even more modest than in table 2 as it ranges from 0.787 to 0.814 instead of 0.779 to 0.818 in table 2. Even though the differences across the two tables for the same specification are significant in a statistical sense, they are economically tiny.

Panel (b) of table 4 duplicates the specifications of panel (b) of table 2 and reports results of regression of instrumented $\log H$ on instrumented $\log K$ that also include a quadratic term for $\log K$. The results indicate the presence of a mild concavity. This is a stronger contrast with table 2 where the results point towards modest convexity. While there is some variation in the estimated coefficient on the quadratic term in $\log K$ across deciles of land area, the average is 0.11. With $\log K$ varying from 11.3 to 12.15 between the bottom and top decile of capital, this concavity implies that the average 'coefficient' on capital ($\log Y/\log K$) is only about 0.1 less at the top decile relative to the bottom decile.

While it is meaningful to rely on the closest center and the distance to it as exogenous predictors of land area, land value, and capital investment, it is useful to experiment with various alternatives. In table 5, we report results for four instrumentation strategies and two different specifications.

Table 5: Housing quantities and capital inputs, regressions using alternate sources of variation for land values

Category Category	centre	dist.	dist.+ centre	dist.+ centre+misc.	centre	dist.	dist.+ centre	dist.+ centre+misc.
$\log(K)$	0.784^a	0.697^a	0.807^a	0.807^a	0.365^a	2.468^a	4.634^a	0.525^a
	(0.00011)	(0.00009)	(0.00017)	(0.00007)	(0.00978)	(0.01350)	(0.01477)	(0.01790)
$[\log(K)]^2$					0.018^a	-0.075^a	-0.161^a	0.012^a
					(0.00041)	(0.00057)	(0.00062)	(0.00075)
R^2	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Observations	8,100	8,100	8,100	8,100	8,100	8,100	8,100	8,100

Notes: OLS regressions with decile fixed effects in all columns. XXXXRobust standard errors in parentheses. *a*, *b*, *c*: significant at 1%, 5%, 10%.

Instead of estimating a separate regression for each decile of land area, we estimate regressions pooling all our observations and impose a fixed effect for each decile of land area. This simplifies the comparison across specifications since each coefficient is estimated only once instead of nine times for nine different deciles.

In column 1 of table 5, we regress the log of housing on the log of capital (and fixed effects for each decile of land area) for all observations located in urban areas as in column 2 of table 3 using only urban area fixed effects as instrument. The estimated coefficient on log capital is very close to those obtained previously at 0.78. In column 2, we only use log distance to the center of the urban area of belonging as instrument. We obtain a lower value of 0.70 for the coefficient on capital. We note that using only distance might be slightly awkward in a cross-city context as being 10 kilometers away from the center of Paris probably commands a much stronger demand for housing than being 10 kilometers away from the centre of a third-tier urban area. In column 3, we use both urban area fixed effects and distance to the center as instruments. This is akin to the approach used in table 4 except that we estimate a single coefficient for capital instead of one for each decile of land area. The estimated coefficient on capital is just below 0.81. This is in the range of the coefficients estimated for each decile in panel (a) of table 4. Finally, in column 4, we also use the control variables mentioned above as excluded instruments. Adding these extra characteristics, which include the degree of completion, the intermediary through which the parcel was purchased, and a dummy for access to services makes no difference to the results relative to the previous column.

Finally, the last four columns of table 5 replicate the first four but add the square of the log of capital as explanatory variable. The results are more fragile. Depending on the exact specification, the log production function is either modestly convex or modestly concave in log capital.

6. Recovering a functional form

So far, we have non-parametrically estimated the production of housing as a function of capital investment given the area of parcels. We then used some regressions to assess the shape of this production function. We reached two main conclusions. As a first approximation, the production function of housing is Cobb-Douglas with a share of capital of about 0.80. However, a more detailed look suggests some log convexity when using the raw data and some log concavity with intrumented values.

In this section, we develop an approach to test a variety of specific functional forms for the production function of housing. Given the precision of our estimates, we can always formally reject any simple functional specification (or less simple specifications, given enough observations). We do not use standard specification tests because they essentially impose arbitrary penalties for more general specifications. Instead, we want to assess how good an approximation a particular functional specification is relative to another. For instance, how much better do we fare by assuming a constant-elasticity-of-substitution (CES) production function relative to a (less general) Cobb-Douglas?

As shown above, measuring the fit of different specifications is unlikely to be very informative. Our estimation of the production of housing relies on kernel-smoothed data for land values and capital inputs. As a result, the R-squared is always very close to unity. We proceed instead as follow. For our exposition to remain concrete, let us consider a CES production function. The production of housing is given by $H = A\left(\alpha K^{(\sigma-1)/\sigma} + (1-\alpha)T^{(\sigma-1)/\sigma}\right)^{\sigma/(\sigma-1)}$ where σ is the elasticity of substitution between land and capital inputs. Using equation (5) and the partial derivative of the CES production function with respect to K, we obtain the following cost share:

$$\frac{rK^*}{rK^* + R(K^*,T)} = \frac{\alpha(K^*)^{1-1/\sigma}}{\alpha(K^*)^{1-1/\sigma} + (1-\alpha)T^{1-1/\sigma}}\,. \tag{13}$$

Using our data, we then estimate α and σ using equation (13) by minimising the sum of the squared distances between actual costs shares and those predicted by a CES production function. We then insert those parameters into the production function and use them to produce counterfactual produced quantities of housing using the actual data about K, R, and T. That is, we estimate the parameters of a CES production function that best fit observed factor shares and then apply this production function to generate counterfactual quantities of housing. We then perform the same regression of $\log H$ on $\log K$, with and without a quadratic term in $\log K$ as in table 2. We can then repeat the same exercise using instrumented values for K, R, and T as in table 4. Aside from the CES, we perform the exercise for the Cobb-Douglas production function and for second- and third-order translog production functions.

Table 6 reports the results for estimations using the observed values of K, R, and T, while table 7 reports the results for estimations using instrumented values of K, R, and T.

In panel (a) of table 6, we can see that generated Cobb-Douglas housing data lead to about the same coefficients as in table 2 but, obviously, fail to replicate the upward trend in this coefficient as higher deciles of land area are considered. The estimated coefficient of 0.783 is the same as the one estimated from factor shares and used to generate the data about housing quantities. Panel (b), which also includes a quadratic term for $\log K$ estimates the same expected coefficient of 0.783 for $\log K$ and a coefficient of 0 for its square, as should also be expected. For the CES case, we note that the estimated parameter values for the production function are $\alpha = 0.783$ and $\sigma = 1.003$. This is extremely close to the Cobb-Douglas case. In panel (c), the coefficients on $\log K$ are, unsurprisingly, very close to those estimated in the Cobb-Douglas except that this panel also replicates the tendency of the capital share to increase as higher deciles of land areas are considered. In panel (d), we nonetheless find that we estimate minimal concavity instead of the convexity reported in panel (b) of table 2. Finding minimum concavity for the generated data is of course consistent with an estimated σ just above one. In panel (e), the second-order translog is also able to replicate the upward trend in the coefficient on $\log K$. Like the CES, these coefficients are nonetheless marginally below those reported in table 2. In panel (f), the second-order translog offers so far the best replication of the results of panel (b) of table 2. The coefficient on $\log K$ is on average very close to that estimated directly from the data and the coefficient on the square of $\log K$ is also about equal to the average value in panel (b) of table 2. Panels (g) and (h) duplicate the last two panels for a third-order translog function instead of a second-order translog. The results match those obtained directly from the data in table 2 very well.

Table 7 repeats the same exercise for instrumented values of K, R, and T instead of the observed values used in table 6. The results should now be compared with those in the two panels of table 4. Again, the Cobb-Douglas data deliver shares of the right magnitude but are unable to generate the

Table 6: Housing quantities and capital inputs fitting specific functional forms, OLS by area decile

Decile	1	2	3	4	5	6	7	8	9
Panel (a): Cobb-Douglas									
$\log(K)$	0.783^a	0.783^a	0.783^a	0.783^a	0.783^a	0.783^a	0.783^a	0.783^a	0.783^a
	(0.00000)	(0.00000)	(0.00000)	(0.00000)	(0.00000)	(0.00000)	(0.00000)	(0.00000)	(0.00000)
Panel (b): Cobb-Douglas									
$\log(K)$	0.783^a	0.783^a	0.783^a	0.783^a	0.783^a	0.783^a	0.783^a	0.783^a	0.783^a
	(0.00000)	(0.00000)	(0.00000)	(0.00000)	(0.00000)	(0.00000)	(0.00000)	(0.00000)	(0.00000)
$[\log(K)]^2$	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	(0.00000)	(0.00000)	(0.00000)	(0.00000)	(0.00000)	(0.00000)	(0.00000)	(0.00000)	(0.00000)
Panel (c): CES									
$\log(K)$	0.770^a	0.775^a	0.778^a	0.781^a	0.784^a	0.786^a	0.787^a	0.789^a	0.790^a
	(0.00005)	(0.00005)	(0.00006)	(0.00006)	(0.00006)	(0.00006)	(0.00006)	(0.00006)	(0.00006)
Panel (d): CES									
$\log(K)$	0.936^a	0.943^a	0.947^a	0.951^a	0.954^a	0.956^a	0.958^a	0.960^a	0.962^a
	(0.00004)	(0.00004)	(0.00004)	(0.00004)	(0.00004)	(0.00004)	(0.00004)	(0.00004)	(0.00004)
$[\log(K)]^2$	-0.007^a	-0.007^a	-0.007^a	-0.007^a	-0.007^a	-0.007^a	-0.007^a	-0.007^a	-0.007^a
	(0.00000)	(0.00000)	(0.00000)	(0.00000)	(0.00000)	(0.00000)	(0.00000)	(0.00000)	(0.00000)
Panel (e): Second-order translog									
$\log(K)$	0.768^a	0.773^a	0.777^a	0.780^a	0.782^a	0.784^a	0.786^a	0.788^a	0.789^a
	(0.00014)	(0.00014)	(0.00014)	(0.00014)	(0.00014)	(0.00014)	(0.00014)	(0.00014)	(0.00014)
Panel (f): Second-order translog									
$\log(K)$	0.332^a	0.337^a	0.341^a	0.343^a	0.346^a	0.348^a	0.350^a	0.351^a	0.353^a
	(0.00000)	(0.00000)	(0.00000)	(0.00000)	(0.00000)	(0.00000)	(0.00000)	(0.00000)	(0.00000)
$[\log(K)]^2$	0.018^a	0.018^a	0.018^a	0.018^a	0.018^a	0.018^a	0.018^a	0.018^a	0.018^a
	(0.00000)	(0.00000)	(0.00000)	(0.00000)	(0.00000)	(0.00000)	(0.00000)	(0.00000)	(0.00000)
Panel (g): Third-order translog									
$\log(K)$	0.776^a	0.777^a	0.779^a	0.781^a	0.784^a	0.787^a	0.790^a	0.793^a	0.795^a
	(0.00022)	(0.00020)	(0.00018)	(0.00017)	(0.00016)	(0.00016)	(0.00015)	(0.00014)	(0.00014)
Panel (h): Third-order translog									
$\log(K)$	0.139^a	0.206^a	0.257^a	0.299^a	0.334^a	0.365^a	0.392^a	0.416^a	0.439^a
	(0.00734)	(0.00734)	(0.00734)	(0.00734)	(0.00734)	(0.00734)	(0.00734)	(0.00734)	(0.00734)
$[\log(K)]^2$	0.027^a	0.024^a	0.022^a	0.020^a	0.019^a	0.018^a	0.017^a	0.016^a	0.015^a
	(0.00031)	(0.00031)	(0.00031)	(0.00031)	(0.00031)	(0.00031)	(0.00031)	(0.00031)	(0.00031)

Notes: OLS regressions with a constant in all columns. 900 observations for each regression. The R^2 is always 1.00 in all specifications. XXXXRobust standard errors in parentheses. *a*, *b*, *c*: significant at 1%, 5%, 10%.

Table 7: Housing quantities and capital inputs fitting specific functional forms and using instrumented values, OLS by area decile

Decile	1	2	3	4	5	6	7	8	9
Panel (a): Cobb-Douglas									
$\log(K)$	0.790^a	0.790^a	0.790^a	0.790^a	0.790^a	0.790^a	0.790^a	0.790^a	0.790^a
	(0.00000)	(0.00000)	(0.00000)	(0.00000)	(0.00000)	(0.00000)	(0.00000)	(0.00000)	(0.00000)
Panel (b): Cobb-Douglas									
$\log(K)$	0.790^a	0.790^a	0.790^a	0.790^a	0.790^a	0.790^a	0.790^a	0.790^a	0.790^a
	(0.00000)	(0.00000)	(0.00000)	(0.00000)	(0.00000)	(0.00000)	(0.00000)	(0.00000)	(0.00000)
$[\log(K)]^2$	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	(0.00000)	(0.00000)	(0.00000)	(0.00000)	(0.00000)	(0.00000)	(0.00000)	(0.00000)	(0.00000)
Panel (c): CES									
$\log(K)$	0.773^a	0.780^a	0.786^a	0.791^a	0.794^a	0.797^a	0.800^a	0.802^a	0.804^a
	(0.00008)	(0.00008)	(0.00008)	(0.00008)	(0.00008)	(0.00008)	(0.00008)	(0.00008)	(0.00008)
Panel (d): CES									
$\log(K)$	1.031^a	1.038^a	1.043^a	1.047^a	1.051^a	1.053^a	1.056^a	1.058^a	1.059^a
	(0.00005)	(0.00005)	(0.00005)	(0.00005)	(0.00005)	(0.00005)	(0.00005)	(0.00005)	(0.00005)
$[\log(K)]^2$	-0.011^a	-0.011^a	-0.011^a	-0.011^a	-0.011^a	-0.011^a	-0.011^a	-0.011^a	-0.011^a
	(0.00000)	(0.00000)	(0.00000)	(0.00000)	(0.00000)	(0.00000)	(0.00000)	(0.00000)	(0.00000)
Panel (e): Second-order translog									
$\log(K)$	0.723^a	0.748^a	0.766^a	0.780^a	0.792^a	0.802^a	0.811^a	0.819^a	0.826^a
	(0.00073)	(0.00073)	(0.00073)	(0.00073)	(0.00073)	(0.00073)	(0.00073)	(0.00073)	(0.00073)
Panel (f): Second-order translog									
$\log(K)$	2.948^a	2.973^a	2.991^a	3.005^a	3.017^a	3.027^a	3.036^a	3.044^a	3.051^a
	(0.00001)	(0.00001)	(0.00001)	(0.00001)	(0.00001)	(0.00001)	(0.00001)	(0.00001)	(0.00001)
$[\log(K)]^2$	-0.094^a	-0.094^a	-0.094^a	-0.094^a	-0.094^a	-0.094^a	-0.094^a	-0.094^a	-0.094^a
	(0.00000)	(0.00000)	(0.00000)	(0.00000)	(0.00000)	(0.00000)	(0.00000)	(0.00000)	(0.00000)
Panel (g): Third-order translog									
$\log(K)$	0.784^a	0.759^a	0.758^a	0.768^a	0.784^a	0.803^a	0.824^a	0.846^a	0.868^a
	(0.00149)	(0.00121)	(0.00099)	(0.00083)	(0.00069)	(0.00057)	(0.00046)	(0.00037)	(0.00030)
Panel (h): Third-order translog									
$\log(K)$	5.335^a	4.434^a	3.781^a	3.271^a	2.855^a	2.505^a	2.202^a	1.937^a	1.701^a
	(0.01200)	(0.01200)	(0.01200)	(0.01200)	(0.01200)	(0.01200)	(0.01200)	(0.01200)	(0.01200)
$[\log(K)]^2$	-0.192^a	-0.155^a	-0.127^a	-0.105^a	-0.087^a	-0.072^a	-0.058^a	-0.046^a	-0.035^a
	(0.00051)	(0.00051)	(0.00051)	(0.00051)	(0.00051)	(0.00051)	(0.00051)	(0.00051)	(0.00051)

Notes: OLS regressions with a constant in all columns. 900 observations for each regression. The R^2 is always 1.00 in all specifications. XXXXRobust standard errors in parentheses. *a*, *b*, *c*: significant at 1%, 5%, 10%.

log concavity estimated in panel (b) of table 4. The CES data are now able to reproduce both the tendency of the capital coefficient to be higher in higher deciles of land area with a linear specification and the estimated log concavity of H in K when adding a quadratic term. We nonetheless note that the amount of concavity obtained from the CES specification is minimal. Again the two translog specifications yield excellent results and are able to match both those from the linear specification of panel (a) of table 4 and those with the addition of a quadratic term in panel (b) of the same table.

[[Paragraph of conclusion to be written: translog is better but Cobb-Douglas is not bad and CES implies an elasticity of substitution extremely close to one]]

7. Full identification?

Sketch:

- In our approach so far, we have assumed that land area is taken as given. Our IV approach allows for the land area to be systematically correlated with unobserved supply characteristics of parcels. For instance, more 'productive' parcels may be systematically smaller (or larger).
- Our approach is also only partially identified in the sense that we can only estimate the production function of housing as a function of capital inputs.
- We can easily extend our approach to allow for profit-maximising developers to optimise profits with respect to parcel area T as well. This introduces a second first-order condition.
- Importantly, for the two first-order conditions to be consistent with zero profit, we need to impose constant returns to scale like Epple *et al.* (2010) and R becomes proportional to T: $R(K,T) = T \times R_2(T)$ which is clearly counterfactual.
- In turn, imposing constant returns to scale would lead us to be able to fully identify the production function of housing.
- To assess whether constant returns to scale is a good assumption, we can proceed as in the previous section 6 and estimate housing quantities under this restriction. We can then regress these alternative generated values of $\log H$ on $\log K$ and on $\log K$ and its square as previously.

- As shown by table 8, the results are not good at all and far from what we estimate with the true data in tables 2 and 4.
- We interpret this as a rejection of constant returns to scale in production even if, again, that may be not be a bad first-order approximation.

Table 8: Housing quantities and capital inputs imposing constant returns to scale in production, OLS by area decile

Decile	1	2	3	4	5	6	7	8	9
Panel (a): raw data									
$\log(K)$	0.768^a	0.755^a	0.743^a	0.733^a	0.726^a	0.721^a	0.717^a	0.715^a	0.712^a
	(0.00013)	(0.00035)	(0.00052)	(0.00068)	(0.00085)	(0.00100)	(0.00115)	(0.00129)	(0.00139)
Panel (b): raw data									
$\log(K)$	0.379^a	-0.216^a	-0.704^a	-1.199^a	-1.736^a	-2.241^a	-2.711^a	-3.121^a	-3.446^a
	(0.00465)	(0.01448)	(0.02162)	(0.02554)	(0.02753)	(0.02796)	(0.02863)	(0.02994)	(0.03261)
$[\log(K)]^2$	0.016^a	0.041^a	0.061^a	0.081^a	0.104^a	0.125^a	0.144^a	0.162^a	0.175^a
	(0.00020)	(0.00061)	(0.00091)	(0.00108)	(0.00116)	(0.00118)	(0.00121)	(0.00126)	(0.00137)
Panel (c): instrumented data									
$\log(K)$	0.787^a	0.811^a	0.833^a	0.868^a	0.897^a	0.907^a	0.911^a	0.913^a	0.928^a
	(0.00036)	(0.00087)	(0.00139)	(0.00180)	(0.00227)	(0.00239)	(0.00238)	(0.00224)	(0.00205)
Panel (d): instrumented data									
$\log(K)$	2.512^a	-4.364^a	-8.040^a	-10.052^a	-11.656^a	-9.737^a	-5.891^a	-3.045^a	-4.566^a
	((0.07447))	(0.14872)	(0.21290)	(0.29771)	(0.42405)	(0.51673)	(0.58163)	(0.57299)	(0.50648)
$[\log(K)]^2$	-0.073^a	0.219^a	0.375^a	0.462^a	0.531^a	0.450^a	0.288^a	0.167^a	0.232^a
	(0.00315)	(0.00629)	(0.00900)	(0.01259)	(0.01793)	(0.02184)	(0.02459)	(0.02422)	(0.02141)

Notes: OLS regressions with a constant in all columns. 900 observations for each regression. The R^2 is always 1.00 in all specifications. XXXXRobust standard errors in parentheses. *a*, *b*, *c*: significant at 1%, 5%, 10%.

8. Conclusions

[[To be written]]

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