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# THE DYNAMICS OF SUBPRIME ADJUSTABLE-RATE MORTGAGE DEFAULT: A STRUCTURAL ESTIMATION

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# The Dynamics of Subprime Adjustable-Rate Mortgage Default: A Structural Estimation*

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**Abstract**

We present a dynamic structural model of subprime adjustable-rate mortgage (ARM) borrowers making payment decisions, taking into account possible consequences of different degrees of delinquency from their lenders. We empirically implement the model using unique data sets that contain information on borrowers' mortgage payment history, their broad balance sheets, and lender responses. Our investigation of the factors that drive borrowers' decisions reveals that subprime ARMs are not all alike. For loans originated in 2004 and 2005, the interest rate resets associated with ARMs as well as the housing and labor market conditions were not as important in borrowers' delinquency decisions as in their decisions to pay off their loans. For loans originated in 2006, interest rate resets, housing price declines, and worsening labor market conditions all contributed importantly to their high delinquency rates. Counterfactual policy simulations reveal that even if the London Interbank Offered Rate (LIBOR) could be lowered to zero by aggressive traditional monetary policies, it would have a limited effect on reducing the delinquency rates. We find that automatic modification mortgages *with cushions,* under which the monthly payment or principal balance reductions are triggered *only when* housing price declines exceed a certain percentage, may result in a Pareto improvement, in that borrowers and lenders are both made better off than under the baseline, with lower delinquency and foreclosure rates. Our counterfactual analysis also suggests that limited commitment power on the part of the lenders regarding loan modification policies may be an important reason for the relatively low rate of modifications observed during the housing crisis.

***Keywords:*** adjustable-rate mortgage, default, loan modification, automatic modification mortgages with cushions

***JEL Classification Codes:*** D12, D14; G2, G21, G33

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# 1 Introduction

The collapse of the subprime residential mortgage market played a crucial role in the housing crisis that subsequently led to the Great Recession.[1] At the end of 2007, subprime mortgages accounted for about 13 percent of all outstanding first-lien residential mortgages but over half of the foreclosures. The majority of the subprime mortgages, both by number and by value, were adjustable-rate mortgages (ARMs); these mortgages had a foreclosure rate of 17 percent, much higher than the 5 percent foreclosure rate for fixed-rate subprime mortgages (Frame, Lehnert, and Prescott, 2008, Table 1). In response to these developments, many government policies were designed and implemented to change the default incentives of the subprime ARM borrowers.[2] Few structural models, however, exist that can guide us in these efforts and that can help us understand why most of the programs had limited success.[3]

In this paper, we develop a dynamic structural model to study the incentives for subprime ARM borrowers to default and investigate how these incentives change under various policies. In our model, at each period, a borrower decides whether to pay the amount due (and be current) or not to pay (and stay in various states of delinquency), taking into account the lender's responses such as mortgage modification, liquidation, or waiting (i.e., doing nothing). Relative to the existing structural models on mortgage defaults, our model has two key distinguishing features: First, default is not the terminal and absorbing state, as we allow borrowers to self-cure their delinquency; second, we consider loan modification as one of the lenders' loss mitigation practices while the existing models only allow for liquidation.

We empirically implement our model using unique mortgage loan-level data sets that contain not only detailed information on borrowers' mortgage payment history and lenders' responses but also credit bureau information about borrowers' broader balance sheets. We are thus one of the first to use borrowers' credit bureau information to understand their mortgage payment decisions.[4] To track movements in the local housing and labor markets, we further merge our data with zip code-level home price indices and county-level unemployment rates.

[1]There is no standard definition of subprime mortgage loans. Typically, they refer to loans made to borrowers with poor credit history (e.g., a FICO score under 620) and/or with a high leverage as measured by either the debt-to-income ratio or the loan-to-value ratio. For the data used in this paper, subprime mortgages are defined as those in private-label mortgage-backed securities marketed as subprime, as in Mayer, Pence, and Sherlund (2009).

[2]To name a few such programs, the FHA Secure program approved by Congress in September 2007; the Hope Now Alliance program created by then-Treasury Secretary Henry Paulson in October 2007; Hope for Homeowners refinancing program passed by Congress in spring 2008; Making Home Affordable initiative in conjunction with the Home Affordable Modification Program (HAMP) and the Home Affordable Refinance Program (HARP) launched by the Obama administration in March 2009. See Gerardi and Li (2010) for more details.

[3]Over the first two and a half years, HARP refinancing activity remained subdued relative to model-based extrapolations from historical experience. From its inception to the end of 2011, 1.1 million mortgages refinanced through HARP, compared with the initial announced goal of 3 million to 4 million mortgages. In December, HARP 2.0 was introduced and HARP refinance volume picked up, reaching 3.2 million by June 2014 (www.fhfa.gov/AboutUS/Reports/Pages/Refinance-Report-February-2014.aspx). Similarly, HAMP was designed to help as many as 4 million borrowers avoid foreclosure by the end of 2012. By February 2010, one year into the program, only 168,708 trial plans had been converted into permanent revisions. Through January 2012, a population of 621,000 loans had received HAMP modifications. See www.treasury.gov/resource-center/economic-policy/Documents/HAMPPrincipalReductionResearchLong070912FINAL.pdf.

[4]Elul, Souleles, Chomsisengphet, Glennon, and Hunt (2010) also use credit bureau information to study mortgage default decisions in their empirical analysis.

Three main factors drive ARM borrowers' mortgage payment decisions: home equity, income, and monthly mortgage payment; importantly, both the current *levels* of these factors *and* the *expectations* of their future changes matter. Borrowers with negative home equity gain little financially from continuing with their mortgage payments, especially when they do not expect housing prices to recover and when costs associated with defaults and foreclosures are low. Changes in incomes and expenses, including changes in monthly mortgage payments because of interest rate resets for example, affect borrowers' liquidity position. In principle, borrowers can refinance their mortgages to lower interest rates or sell their houses to improve their liquidity positions, but these options may not be available in the presence of declining housing prices, increasing unemployment rates, and/or tightened lending standards. These constrained borrowers thus may find it optimal to default on their mortgages despite the possible consequences of foreclosure.

To quantify the relative importance of these different drivers of default, we simulate our structurally estimated model under various counterfactual scenarios. Our simulation results suggest that the factors that drive borrower delinquency and foreclosures differ substantially by loan origination year. For loans originated in 2004 and 2005, which preceded the severe downturn of the housing and labor markets, the interest rate resets associated with ARMs as well as the housing and labor market conditions do not seem to be as important factors for borrowers' delinquency behavior as they are in determining whether the borrowers would pay off their loans (i.e., sell their houses or refinance). However, for loans that originated in 2006, interest rate reset, housing price declines, and worsening labor market conditions all contributed to their high delinquency rates, with housing price declines being the most significant contributing factor.[5] These results arise because, for loans originated in 2004 and 2005, interest rates did not reset until 2006 or 2007, at which time housing prices had just begun to decline. More importantly, because housing prices continued to appreciate in 2004, 2005, and part of 2006, these borrowers had accumulated some home equity by the time of their interest rates reset; in fact, in many places housing prices did not drop to their 2004 levels until 2008. Additionally, the labor market did not deteriorate significantly until 2008 or 2009. In contrast, borrowers whose loans originated in 2006 had the perfect storm in 2008 or 2009 when their interest rates reset, as housing prices had depreciated substantially and unemployment rates had risen sharply.

Counterfactual policy simulations reveal that even if the LIBOR rate could be lowered to zero by aggressive traditional monetary policies, it would have a limited effect on reducing the delinquency rates. We find that automatic modification mortgage designs under which the monthly payments or the principal balances of the loans are automatically reduced when housing prices decline can be effective in reducing both delinquency and foreclosure. Importantly, we find that automatic modification mortgages *with a cushion,* under which the monthly payment or principal balance reductions are triggered *only when* housing price declines exceed a certain percentage, may result in a *Pareto improvement* in that borrowers and lenders are both made better off than under the

[5]Our finding is consistent with those in the literature including Bhutta, Dokko, and Shan (2010); Foote, Gerardi, and Willen (2012); and Fuster and Willen (2015). Bhutta, Dokko, and Shan (2010) also find that 80 percent of the defaults in their sample (2006 loans originated in the crisis states) are the results of income shocks combined with negative house equity. Foote, Gerardi, and Willen (2012) find that interest rate reset raised the default rates of 2006 loans.

baseline, with lower delinquency and foreclosure rates. Our counterfactual analysis also suggests that limited commitment power on the part of the lenders regarding loan modification policies may be an important reason for the relatively low rate of modifications observed during the housing crisis.

There are several structural models on mortgage defaults and foreclosures. The work of Bajari, Chu, Nekipelov, and Park (2013) is most related to our paper both in questions addressed and in the empirical methodology. However, there are several key differences. First, we incorporate mortgage modification as a possible lender response, while they do not. Second, we allow for borrowers to self-cure, while they treat default as a terminal event that leads to liquidation with certainty.[6] Third, we focus on subprime ARMs, which were much more prevalent than the subprime fixed-rate mortgages that they focus on. Fourth, the two papers differ in the way the effect of counterfactual policies is examined. These differences enable us to study the effects of exogenously changing lenders' actions on borrowers' behavior and to shed light on why lenders were not willing to modify loans. More importantly, the effects of alternative policies such as automatic modification mortgages with cushions can be studied in our framework because this involves changing borrowers' expectations about the co-evolution of housing prices, mortgage balances, and payment sizes.

Campbell and Cocco (2015) study a dynamic model of households' mortgage decisions incorporating labor income, housing prices, inflation, and interest rate risk to quantify the effects of adjustable versus fixed mortgage rates, mortgage loan-to-value ratio, and mortgage affordability measures on mortgage premia and default. Corbae and Quintin (2015) solve an equilibrium model to evaluate the extent to which low down payments and interest-only mortgages were responsible for the increase in foreclosures in the late 2000s. Garriga and Schlagenhauf (2008) study the effects of leverage on default using long-term mortgage contracts. Hatchondo, Martinez, and Sanchez (2011) investigate the effect of broader recourse on default rates and welfare. Mitman (2012) considers the interaction of recourse and bankruptcy on mortgage defaults. Chatterjee and Eyigungor (2015) analyze the default of long-duration collateralized debt. None of these papers make use of mortgage loan-level data as in our paper and in Bajari et al. (2013).

There are also several recent empirical papers that use regression analyses to study lenders' loss mitigation practices and the impact of government intervention policies on these practices. For example, Haughwout, Okah, and Tracy (2010) estimate a competing risk model using modifications (excluding capitalization modifications) of subprime loans that were originated between December 2004 and March 2009. They find a substantial impact of payment reduction on mortgage redefault rates. Agarwal et al. (2011) analyze lenders' loss mitigation practices including liquidation, repayment plans, loan modification, and refinance of mortgages that originated between October 2007 and May 2009 using Mortgage Metrics data from Office of the Comptroller of the Currency (OCC) and Office of Thrift Supervision (OTS), and find a much more modest effect of mortgage modification on defaults. In a subsequent paper, Agarwal et al. (2015) study the impact of the 2009 Home Affordable Modification Program on lenders' incentives to renegotiate mortgages.

Finally, our paper also adds to the growing literature on the recent subprime mortgage crisis,

[6] Adelino, Gerardi, and Willen (2013) show the importance of self-cure as a hindrance for loan modifications.

including, among many others, Foote, Gerardi, and Willen (2008); Gerardi et al. (2008); Keys et al. (2010); and Demyanyk and van Hemert (2011). Additionally, Piskorski, Seru, and Vig (2010) find that securitization reduced mortgage renegotiations and led to more foreclosures. In contrast, Adelino, Gerardi, and Willen (2013) show that it is information asymmetries rather than securitization that hindered mortgage renegotiations.

The remainder of the paper is organized as follows. In Section 2, we describe the data sets we use in our empirical analysis and present the descriptive statistics. In Section 3, we present our model of borrowers' behavior and their interactions with the lenders in a stochastic environment with shocks to housing prices, unemployment rates, and LIBOR rates. In Section 4, we briefly discuss how we solve and estimate our model. In Section 5, we present our estimation results. In Section 6, we describe the goodness-of-fit between the predictions of our model under the estimated parameters and their data analogs. In Section 7, we present results from several counterfactual experiments. In Section 8, we conclude and discuss avenues for future research.

# 2 Data

## 2.1 Data Source

Our data on mortgages and their modifications come from three different sources: the CoreLogic Private Label Securities data – ABS; the CoreLogic Loan Modification data; and the TransUnion Consumer Risk Indicators for Non-Agency RMBS data (also known as "TransUnion-CoreLogic Credit Match Data"). The CoreLogic ABS data consist of loans that were originated as subprime and Alt-A loans and represent about 90 percent of the market. The data include loan-level attributes generally required of issuers of these securities when they originate the loans as well as their historical performance, which are updated monthly. The attributes include borrower characteristics (credit scores, owner occupancy, documentation type, and loan purpose), collateral characteristics (mortgage loan-to-value ratio, property type, zip code), and loan characteristics (product type, loan balance, and loan status).

The CoreLogic Loan Modification data contain information on modifications on loans in the CoreLogic ABS data. The data include detailed information about modification terms, including whether the new loan has a fixed interest rate, the new interest rate, whether some principal is forgiven, whether the mortgage term is changed, etc. Merging the two data sets is straightforward, as each loan is uniquely identified by the same loan ID in both data sets.

The TransUnion Consumer Risk Indicators for Non-Agency RMBS data provide consumer credit information from TransUnion for matched mortgage loans in CoreLogic's private label securities databases. TransUnion employs a proprietary match algorithm to link loans from the CoreLogic databases to borrowers from TransUnion credit repository databases, allowing us to access many borrower-level consumer risk indicator variables, including borrowers' credit score and income at origination, among many others.

We then merge our data with CoreLogic monthly zip code-level repeat-sales housing price index and county-level unemployment rates from the Bureau of Labor Statistics. Thus, our constructed data have several advantages over most of those used in the literature. First, the match with the mortgage modification data allows us to accurately identify lenders' actions and separate delinquent mortgages that are self-cured from delinquent mortgages that become current after lender modification. Second, the TransUnion data enable us to capture borrowers' other liabilities as well as the payment history of these liabilities as summarized by credit scores, which are important for borrowers' mortgage payment decisions.

## 2.2 Mortgage Loans: Summary Statistics

We focus on subprime ARM loans originated in four major housing crisis states (Arizona, California, Florida, and Nevada) between 2004 and 2007.[7] In particular, we take a 1.75 percent random sample of ARMs with initial fixed interest rates for a period of two or three years and a mortgage maturity of 30 years, which are for borrowers' primary residences, are first liens, and not guaranteed by government agencies such as Fannie Mae, Freddie Mac, the Federal Housing Administration (FHA), and the Veterans Administration (VA). We follow these loans until February 2009, before the first coordinated large-scale government effort to modify mortgage loans, the Making Home Afford-able program, was unveiled. In total, we have 16,347 mortgages and 337,811 monthly observations. Of the 16,347 mortgages, 11 percent were originated in Arizona, 55 percent in California, 28 percent in Florida, and 6 percent in Nevada. Not surprisingly, the largest percentage of the loans were originated in 2005 (43 percent), followed by 2004 (37 percent), 2006 (17 percent), and 2007 (2 percent).

Table 1 provides summary statistics of the mortgage loans at origination (at the loan level) and of the whole dynamic sample period (at the loan-month level). The average age of the loan is 16 months in the sample, and the median is 14 months. At origination, 81 percent of the sample are loans with two-year fixed rates. Through the sample period, however, 76 percent of the sample are loans originated with two-year initial fixed rate periods, indicating that more of those loans have terminated via payoff/refinance or foreclosure. More than 90 percent of the loans have prepayment penalties. About 40 percent of the mortgages at origination are interest-only mortgages, and the percentage becomes slightly higher in the whole dynamic sample. About half of the mortgages have full documentation both at origination and through the sample period. While 43 percent of the mortgages are purchase loans at the origination, the ratio increases to 48 percent in the whole dynamic sample. Consistent with being subprime, mortgage borrowers in the sample all have relatively low credit scores, averaging 445 at origination; the scores deteriorate somewhat as the loans age.[8] Additionally, both the average and the median mortgage loan-to-value ratios at origination are around 80 percent, and they do not change much as the loans age. The annual household income estimated by TransUnion averages $72,000 at origination with a median of $67,000. Loan balances average $259,000 at origination with a median of $228,000. These numbers are not very different from their dynamic counterparts. The mortgage interest rates average 7.13 percent at origination with a median of 6.99 percent. Dynamically, both the mean and median

[7]The subprime mortgage market dried up after 2007.

[8]The credit scores are estimated by TransUnion. They range between 150 and 950, with a high score indicating low risk.

| | At Origination | | | Dynamic Sample | | |
|---|---|---|---|---|---|---|
| Variable | Mean | Median | Std. Dev. | Mean | Median | Std. Dev. |
| Age of the loan (months) | 0 | 0 | 0 | 16 | 14 | 11 |
| Share of 2-year fixed period (%) | 81 | 1 | 39 | 76 | 1 | 41 |
| Prepayment penalty (%) | 0.90 | 1 | 0.30 | 0.92 | 1 | 0.27 |
| Interest-only mortgages (%) | 40 | 0 | 49 | 44 | 0 | 50 |
| Full document at origination (%) | 52 | 1 | 50 | 52 | 1 | 50 |
| Purchase loan (%) | 43 | 0 | 50 | 48 | 0 | 50 |
| Credit score | 445 | 445 | 155 | 424 | 432 | 178 |
| LTV ratio at origination (%) | 79 | 80 | 11 | 81 | 78 | 21 |
| Annual income ($1,000) | 72 | 67 | 26 | | | |
| Principal balance ($1,000) | 259 | 228 | 141 | 260 | 228 | 141 |
| Current interest rate (%) | 7.13 | 6.99 | 1.15 | 7.35 | 7.13 | 1.39 |
| Remaining mortgage terms (months) | 360 | 360 | 0 | 345 | 347 | 11 |
| Monthly payment ($1,000) | 1.616 | 1.429 | 0.859 | 1.679 | 1.475 | 0.902 |
| Maximum lifetime interest rate (%) | 13.50 | 13.45 | 1.28 | 13.42 | 13.38 | 1.27 |
| Minimum lifetime interest rate (%) | 6.70 | 6.89 | 1.86 | 6.59 | 6.85 | 1.90 |
| Periodic interest rate cap (%) | 1.20 | 1.00 | 0.33 | 1.20 | 1.00 | 0.32 |
| Periodic interest rate floor (%) | 0.01 | 0 | 0.13 | 0.01 | 0 | 0.13 |
| First interest rate cap (%) | 2.50 | 3.00 | 0.87 | 2.53 | 3 | 0.91 |
| Margin for adjustable-rate loans (%) | 5.74 | 5.95 | 1.17 | 5.67 | 5.90 | 1.21 |
| 30 days delinquent (%) | 0 | 0 | 0 | 6.86 | 0.0 | 25.37 |
| 60 days delinquent (%) | 0 | 0 | 0 | 3.10 | 0.0 | 17.33 |
| 90 days delinquent (%) | 0 | 0 | 0 | 1.62 | 0.0 | 12.63 |
| 120 days delinquent (%) | 0 | 0 | 0 | 1.40 | 0.0 | 11.73 |
| 150 days delinquent (%) | 0 | 0 | 0 | 1.25 | 0.0 | 11.11 |
| 180 days delinquent (%) | 0 | 0 | 0 | 1.14 | 0.0 | 10.63 |
| 180 days more delinquent (%) | 0 | 0 | 0 | 3.86 | 0.0 | 19.27 |
| House liquidation (%) | 0 | 0 | 0 | 0.64 | 0.0 | 8.08 |
| Loan modification (%) | 0 | 0 | 0 | 0.26 | 0.0 | 5.06 |
| Deviation local unemployment rates (%) | | | | -1.51 | -1.81 | 1.40 |
| Local housing price growth rates (%) | | | | -0.32 | -0.27 | 2.15 |
| Number of observations | | 16,347 | | | 337,811 | |

Table 1: Summary Statistics of Selected Mortgage Loans

mortgage interest rates are higher by 20 and 15 basis points, respectively, as many of these ARMs reset to higher rates after the initial fixed-rate periods expire. The ARMs in our data have a lifetime maximum interest rate of 13.50 percent on average at origination, close to the dynamic average of 13.42 percent; and the lifetime minimum interest rate averages 6.7 percent at origination and 6.59 percent in the dynamic sample. The margin above LIBOR rate when interest rates are adjusted averages 5.74 percent at origination and 5.67 percent in the dynamic sample. Both at origination and in the dynamic sample, the period interest rate adjustment has a cap of 1.2 percent and a floor of 0.01 percent on average. The first interest rate adjustment cap, however, is higher at 2.5 percent on average at origination and 2.53 percent in the dynamic sample. Unemployment rates tend to be lower than their recent local historical averages. Local housing prices, on the other hand, all depreciate in our sample period.

Two observations emerge from Table 1. First, some mortgages stay in delinquency status for a long time without being liquidated. Particularly, in our loan-month dynamic sample, close to 7 percent of loans are 30 days delinquent, 3 percent are 60 days delinquent, and 2 percent are 90 days delinquent. Almost 4 percent of the loans are delinquent for more than half a year. The liquidation rate, in contrast, is only 0.64 percent if measured at loan-month level.[9] Of course, at the loan level, 2,177 out of the 16,347 loans in our random sample were liquidated (see Table 2), resulting in a 13.3 percent foreclosure rate, similar to what others have documented in the literature. Second, at the loan-month level, about 0.26 percent of all mortgage loans are modified by their lenders. This ratio is obviously much higher if we condition on loans that are delinquent. At the loan level, 857 out of the 16,347 loans in our randomly selected sample were modified, resulting in a modification rate of about 5.24 percent. We elaborate on the second observation regarding lenders' decisions in more detail in the next subsection.

In the Appendix, we provide summary statistics of the mortgage loans separately by the origination year, both at the time of origination and over time in Tables A1 to A3. As can be seen, the loans originated in later years are riskier, more likely to have two-year interest fixed periods instead of three-year, more likely to be interest-only mortgages, less likely to have full documentation, and more likely to be purchase loans instead of refinance loans. Their principals, the initial interest rates, and monthly payments are also larger. Furthermore, the maximum and minimum lifetime interest rates and margins have risen over time. Finally, mortgage delinquency rates are much higher for loans originated in later years than in earlier years.

### 2.3 Lenders' Choices: Descriptive Statistics

From Table 1, we observe that lenders do not always respond to borrowers' mortgage delinquencies immediately by liquidating them. In this subsection, we describe lenders' decisions in more detail.

Table 2 presents the delinquency status (in months) *at the beginning of the month* when the loans were liquidated and modified. It shows that mortgage liquidation typically occurs when the

[9]House foreclosure can be a long and expensive process, especially in states with judicial foreclosure laws (Li 2009). Of the four states that we study, Florida requires judicial foreclosure. Arizona, California, and Nevada allow for both judicial and nonjudicial foreclosures, but most are nonjudicial.

| Begnning-of-the-Month Loan Status | At Liquidation (%) | At Modification (%) |
|---|---|---|
| Current | 0.00 | 17.09 |
| 1 month | 0.05 | 18.71 |
| 2 months | 0.05 | 10.74 |
| 3 months | 0.87 | 8.55 |
| 4 months | 2.39 | 6.12 |
| 5 months | 2.71 | 7.39 |
| 6 months | 10.98 | 4.62 |
| 7 months | 26.32 | 4.50 |
| 8 months | 15.48 | 5.31 |
| 9 months | 9.00 | 4.04 |
| 10 months | 7.35 | 2.54 |
| 11 months | 5.19 | 1.96 |
| 12 months | 3.81 | 1.50 |
| 13 months | 4.04 | 0.92 |
| 14 months | 3.12 | 1.73 |
| 15 months | 2.02 | 1.15 |
| 16 months | 2.07 | 0.81 |
| More than 17 months | 4.46 | 2.31 |
| Number of observations | 2,177 | 857 |

Table 2: Loan Status at the Beginning of the Month when Liquidation or Modification Occurs

borrowers were between six and nine months delinquent. While houses with loans less than three months delinquent are rarely liquidated, many houses are liquidated when the mortgages are more than one year delinquent; indeed, about 4.46 percent of the loans liquidated are more than 17 months delinquent. As a side note, the average loan age at liquidation is 27 months; about half of the liquidations occurred in 2008, 30 percent in 2007, 8 percent in 2006, and about 6 percent in the first two months of 2009.

Loan modifications are offered generally to loans already in distress. Nearly 60 percent of the loans are three months or more behind payments at the time of modification. Close to 9 percent are one year or more behind on payments. What is interesting, however, is that about 17 percent of the loans are modified when they are listed as current at the beginning of the period. The majority of these loans (55 percent) are originated in 2005 and the rest mostly in 2006 (37 percent). Furthermore, the majority of the modifications occur within three months of interest rate reset.[10]

Table 3 presents the modification terms. The majority of the modifications result in more affordable mortgages, as 83 percent have a reduction in monthly payments of about $542 on average. However, 8.6 percent of the modifications result in higher payments of about $287 on average, and 8 percent of the modifications lead to less than $50 of monthly payment changes. Capitalization in modification is very common, with arrears added to the principal balance. Indeed, more than 64 percent of the modified loans have increases of principal balances, averaging $12,248. About 30 percent of the modified loans experience less than $500 in the change of principal balances, and

[10]Haughwout, Okah and Tracy (2010) documented similar observations, but their sample is different from ours because they include fixed-rate mortgages, ARMs that have more than three years of fixed interest rate period, and mortgages with maturity not equal to 30 years (Table 3).

| Variable | Reduction | No Change* | Increase |
|---|---|---|---|
| Monthly payment (percentage) | 83.41 | 7.95 | 8.64 |
| Average change in monthly payment ($) | -542 | 1 | 287 |
| | (443) | (19) | (1,141) |
| Balance (percentage) | 5.41 | 30.18 | 64.40 |
| Average change in balance ($) | -34,030 | -73 | 12,248 |
| | (39,603) | (143) | (11,993) |
| Interest rate (percentage) | 83.11 | 16.89 | 0.00 |
| Average change in interest rate (percentage) | -2.980 | 0.00 | |
| | (1.415) | (0.00) | |

Table 3: Terms of Modification

Notes: *No change* refers to changes in monthly payment of less than $50 or total loan balance change of less than $500. Standard deviations are in parentheses.

only 5.4 percent of the loans have principal reductions averaging $34,030.[11] Nonetheless, more than 83 percent of the modified loans have an annualized interest rate reduction averaging 2.98 percent, leading to reduced monthly payments. No modified loans experience interest rate increases. All the loans are made current after modification.

# 3 The Model

In this section, we present a model of a borrower's behavior from the time his mortgage is originated until period $T$, which we specify later. We do *not* endogenously model lenders' decisions in this paper; instead, we estimate them parametrically from the data. We assume that borrowers take lenders' decisions as given.

Time is discrete, denoted by $t = 1, 2, ..., T$, with each period representing *one month*. We use $x_t$ to denote the borrower's state vector in period $t$, which includes time-invariant borrower and mortgage characteristics (e.g., information collected at mortgage origination and house location) as well as time-varying characteristics (e.g., a mortgage's delinquency status, interest rates, local housing market conditions, local unemployment rates.).

## 3.1 Choice Set

In each period $t$, after information $x_t$ is realized, a borrower chooses an action $j$. He has three choices: make the monthly mortgage payment, skip the payment, or pay off the mortgage (which we denote by "PO"). We assume that the option to pay off the mortgage is available to any borrower, regardless of the delinquency status.[12,13]

[11] See Section 3.3 for how we model the lenders' terms of loan modification in our empirical analysis.

[12] In the data, about 86 percent of those who paid off loans were current in their mortgages at the time of the payoffs, and 9 percent, 2 percent and 1 percent were one, two, and three months delinquent, respectively. Very few mortgage payoffs were by borrowers who were more than three months delinquent. Our conversation with the industry experts suggests that because of information delay, borrowers who have chosen to prepay may sometimes be recorded as having a one-month delay.

[13] In reality, a borrower can pay off the mortgage by refinancing or by selling the house. Our data, unfortunately, do not allow us to make such a distinction.

More specifically, a borrower has different options of making mortgage payments, depending on the number of late monthly payments he has, which we denote by $d$ where $d \geq 0$. If the borrower is current on his mortgage payment (i.e., $d = 0$), then he decides whether to make one monthly payment, which we denote by $P_t$ and specify it in Equation (2), to miss the payment, or to pay off the loan.[14] If the borrower is one month behind on the payment (i.e., $d = 1$), then he can choose to pay just $P_t$ and remain one month delinquent, pay $2P_t$ to bring his status to current again,[15] to miss the payment again and thus his status will be $d = 2$ next period; or to pay off the loan. In general, therefore, if a borrower has $d \geq 2$ unpaid monthly payments at the beginning of time $t$, he can choose to make payments of 0, $P_t, 2P_t, \cdots, (d+1)P_t$ or pay off the loan. However, we simplify the problem by assuming that, for $d \geq 2$, if the borrower decides to pay he only has the options to pay 0, $(d-1)P_t, dP_t$, or $(d+1)P_t$ to become $(d+1)$-month delinquent, two months delinquent, one month delinquent, or current, respectively, or to pay off the loan.[16] Formally, a borrower's choice set with $d$ unpaid payments is denoted by $J(d)$, and given by:

$$J(d) = \begin{cases} \{0, 1, \mathrm{PO}\}, & \text{if } d = 0; \\ \{0, 1, 2, \mathrm{PO}\}, & \text{if } d = 1; \\ \{0, d-1, d, d+1, \mathrm{PO}\}, & \text{if } d \geq 2, \end{cases}$$

where choice 0 refers to the action of not making any payment and "PO" refers to paying off the loan. In the remainder of the paper, we sometimes denote the choice set by $J(x_t)$ instead of $J(d)$ because $x_t$ includes the loan delinquency status $d$. We denote the borrower's chosen number of payments in period $t$ as $n_t \in J(d_t)$.

## 3.2 State Transition

The evolution of the state variables is captured by the transition probability $F(x_{t+1}|x_t, j)$, where, as we discussed previously, $x_t$ represents the state vector and $j \in J(x_t)$ represents the borrower's action at time $t$. We now discuss each of the state variables.

**Interest Rate, Monthly Payment, Mortgage Balance, and Liquidation.** A mortgage contract with adjustable rates specifies the initial interest rate, the length of the period during which the initial rate is fixed, mortgage maturity, the rate to which the mortgage rate is indexed, the margin rate, the frequency at which the interest rate is reset, the cap on interest rate change in each period, and the mortgage lifetime interest rate cap and floor. As stated in Section 2, we focus on loans that have two or three years fixed-interest rates and 30 years maturity. Almost all of the

[14] Given that we model the behavior of an ARM borrower, a monthly payment is potentially time-varying, which is reflected in the time subscript in $P_t$.

[15] We do not observe penalty directly in the data. In the model, we allow for different payoff for each decision, which potentially captures the disutility from penalty associated with missing payments; see Subsection 3.4 for more details.

[16] In the data, we do not observe borrower's payment decisions directly. Instead, we observe their loan status. In our sample, once a loan becomes $d \geq 2$ months delinquent, we do not observe that its delinquency status goes down yet still leaves the borrower three or more months delinquent.

loans have six-month adjustment frequencies after the initial fixed period.

We now describe how the interest rate evolves through the life of an ARM loan contract. Let $i_0$ denote the initial interest rate and let $i_r$ denote the new mortgage interest rate at the $r$-th reset. For example, $i_1$ denotes the interest rate at the first reset right after the fixed-rate period. The term MARGIN represents the *margin rate*, which is the margin above the index rate that the new interest will be reset to. All ARMs in our selected sample data are indexed to the six-month LIBOR rate; we use $\text{LIBOR}_t$ to denote the index rate at time $t$. An ARM contract also specifies a *lifetime interest rate floor* and a *lifetime interest rate cap,* which we denote by LFLOOR and LCAP, respectively. The ARM interest rate is restricted to be within the band specified by LFLOOR and LCAP even though MARGIN above the LIBOR rate may go outside the band. ARM loan contracts also specify a cap on the permissible interest rate adjustment in each period, which we denote by PCAP; moreover, for most mortgages, the cap on interest rate change for the first reset at the end of the initial fixed rate is different from the subsequent caps; we thus denote the cap on the interest rate change at the first reset by FCAP.[17] Combining all the elements, the new interest rate at the $r$-th reset in period $t_{(r)}$ evolves as follows:

$$i_r = \begin{cases} \max\left\{i_{r-1} - \text{FCAP}, \text{LFLOOR}, \min\left\{\text{MARGIN} + \text{LIBOR}_{t_{(r)}-1}, i_{r-1} + \text{FCAP}, \text{LCAP}\right\}\right\}, & \text{if } r = 1; \\ \max\left\{i_{r-1} - \text{PCAP}, \text{LFLOOR}, \min\left\{\text{MARGIN} + \text{LIBOR}_{t_{(r)}-1}, i_{r-1} + \text{PCAP}, \text{LCAP}\right\}\right\}, & \text{if } r > 1, \end{cases} \tag{1}$$

where the first term in equation (1) is the lowest interest rate the mortgage can have, assuming the periodic interest change takes its maximum allowed value, the second term is the lowest lifetime interest rate the mortgage can have, and the third term is the lowest of three rates: LIBOR rate plus margin, last period interest rate plus the maximum allowed periodic interest adjustment, lifetime mortgage interest rate cap. Note that $\text{LIBOR}_{t_{(r)}}$ evolves stochastically. The borrower, therefore, needs to form expectations about future values for LIBOR in order to predict the interest rate he will have to pay. The values for the other mortgage parameters, $\{$MARGIN,LFLOOR,LCAP,FCAP,PCAP$\}$ , are fixed throughout the life of the mortgage.

It follows from equation (1) that $i_r \in [\max\{i_{r-1} - \text{FCAP}, \text{LFLOOR}\}, \min\{i_{r-1} + \text{FCAP}, \text{LCAP}\}]$ if $r = 1$ and that $i_r \in [\max\{i_{r-1} - \text{PCAP}, \text{LFLOOR}\}, \min\{i_{r-1} + \text{PCAP}, \text{LCAP}\}]$ if $r > 1$. In other words, $\{$LFLOOR,LCAP,FCAP,PCAP$\}$ put bounds on the volatility of the ARM interest rate: even when LIBOR is very volatile, the mortgage interest rate may not change significantly if FCAP, PCAP, and LCAP $-$ LFLOOR are low.

Given the rule that determines the interest rate reset, we now specify the transition of an ARM interest rate from period $t$ to period $t + 1$. With a slight abuse of notation, let $r(t)$ denote the number of resets that occurred up to period $t$.[18] Note that either $r(t+1) = r(t)$ or $r(t+1) = r(t)+1$. The former is true when both period $t$ and $t + 1$ are in between two resets, hence $i_{r(t+1)} = i_{r(t)}$. The latter is true when an interest rate is just reset in period $t + 1$, hence $i_{r(t+1)} = i_{r(t)+1}$, where

[17] Typically, FCAP is larger than PCAP; that is, the interest rate change is typically larger at the initial reset than at subsequent resets.

[18] For example, if the initial interest rate is fixed for at least $t$ periods, then $r(t) = 0$. If an interest rate is reset for the second time in period $t$, $r(t) = 2$.

$i_{r(t)+1}$ is calculated using the formula in (1).

Once the new interest rate is determined, the new monthly payment can be calculated based on the interest rate and the beginning-of-the-period mortgage balance. Consider a borrower in period $t$ with remaining mortgage balance $\text{BAL}_{t-1}$ and interest rate $i_{r(t)}$. The borrower's monthly mortgage payment $P_t$ is calculated so that if the borrower makes a fixed payment of $P_t$ until the 360th period (i.e., the end of the 30-year loan term), he will pay off the entire mortgage; specifically,

$$P_t = \frac{\text{BAL}_{t-1} \times i_{r(t)} / 12}{1 - \left(1 + i_{r(t)} / 12\right)^{-(360-t+1)}}, \tag{2}$$

and the new balance entering period $t+1$ is updated to:

$$\text{BAL}_t = \text{BAL}_{t-1} \times \left[1 - \frac{i_{r(t)} / 12}{\left(1 + i_{r(t)} / 12\right)^{360-t+1} - 1}\right]. \tag{3}$$

**Remark 1.** *Note that the lender's decisions affect the transition of the borrower's state variables, i.e., $F(x_{t+1}|x_t, j)$ incorporates the lender's responses. If the lender chooses to modify the loan, it will lead to possible changes to the borrower's loan status, interest rate, monthly payment, and mortgage balance. We describe how modification affects the mortgage balance, interest rate, monthly payment, and loan status in Section 3.3. If the lender chooses to liquidate the house, then the borrower will be forced into the state of liquidation.*

**Other State Variables.** Other state variables include the number of late monthly payments $d_t$, the LIBOR rate $\text{LIBOR}_t$, housing price $h_t$, changes in local unemployment rate relative to its trend $\Delta\text{UNR}_t$, and borrower credit score $\text{CS}_t$. The evolution of these state variables is as follows:

- *Number of late monthly payments ($d_t$):* $d_{t+1} = d_t - n_t + 1$, where $n_t \in J(d_t)$ is the number of monthly payments a borrower makes at time $t$.

- *LIBOR rates* ($\text{LIBOR}_t$): We assume that the borrower's belief regarding the evolution of LIBOR rates is that it follows an AR(1) process in logs.

$$\ln(\text{LIBOR}_{t+1}) = \lambda_0 + \lambda_1 \ln(\text{LIBOR}_t) + \epsilon_{\text{LIBOR},t},$$

  where $\epsilon_{\text{LIBOR},t} \sim N(0, \sigma^2_{\text{LIBOR}})$ is assumed to be serially independent.

- *Housing price* ($h_t$)**:** We assume that the borrower's belief regarding the evolution of housing prices in each zip code is that it follows an AR(1) process:

$$h_{t+1} = \lambda_2 + \lambda_3 h_t + \epsilon_{h,t},$$

  where $\epsilon_{h,t} \sim N(0, \sigma^2_h)$ is assumed to be serially independent.

- *Local unemployment rate (* $\Delta\text{UNR}_t$): We focus on the deviation of the current unemployment rate $\text{UNR}_t$ in a *county* from the average of monthly unemployment rates from 2000 to 2009 in the same county $\overline{\text{UNR}}$, which we denote by $\Delta\text{UNR}_t = \text{UNR}_t - \overline{\text{UNR}}$. We assume that the borrower's belief regarding the evolution of $\Delta$UNR is that it follows an AR(1) process:

$$\Delta\text{UNR}_{t+1} = \lambda_4 + \lambda_5 \Delta\text{UNR}_t + \epsilon_{\text{UNR},t},$$

where $\epsilon_{\text{UNR},t} \sim N(0, \sigma^2_{\Delta\text{UNR}})$ is assumed to be serially independent.

- *Credit score* ($\text{CS}_t$): We assume that the borrower's belief regarding the evolution of the log of his credit score is that it has the following process:

$$\ln(\text{CS}_{t+1}) = \lambda_6 + \lambda_7 \ln(\text{CS}_t) + \lambda_8 1[d_t = 1] + \lambda_9 1[d_t = 2] + \lambda_{10} 1[d_t = 3] + \lambda_{11} 1[d_t \geq 4] + \epsilon_{\text{CS},t},$$

where $1(\cdot)$ is the indicator function and $\epsilon_{\text{CS},t} \sim N(0, \sigma^2_{\text{CS}})$ is assumed to be serially independent.

## 3.3 Loan Modification and Foreclosure

A lender considers the following decisions each period: *foreclose the house*, *modify the loan*, or *wait* (i.e., do nothing). As we mentioned in the introduction, in this paper we do not endogenize these decisions; rather, we assume that lenders follow decision rules that depend on borrowers' various characteristics and are invariant to policy changes.[19] Borrowers take these decision rules as given.

As we describe in detail in Section 5.1, we specify that the probability that the lenders will choose one of the three options depends on the delinquency status and a rich set of loan and housing characteristics. We estimate these lender decision rules by flexible logit or multinomial logit regressions.

If the lender chooses to foreclose the house, the borrower receives the payoff associated with liquidation (see eq. (6)). If the lender chooses to wait, then the borrower's terms of the loan stay unchanged. However, if the lender chooses to modify the loan, we need to specify the new terms of the modified loan. Here we recall from Table 3 in Section 2 that the most popular modification is recapitalization coupled with interest rate reset. Ideally, we would like to estimate the bi-dimensional lender's choice of the new balance and new interest rate of the modified loan; however, instead of estimating such a joint process, we assume for simplicity that the new term of the modified loan is determined as follows:

- After modification, the borrower's payment status is made current, i.e., $d_{t+1} = 0$;
- The new balance upon modification will be the sum of the premodification loan balance and

[19] This characterization of lender behavior is consistent with the data. In a companion paper, we endogenize lenders' decisions and investigate why they did not respond to the various policies introduced by the government to reduce foreclosures and encourage loan modifications.

the arrears in late payments, i.e.,[20]

$$\text{BAL}_{t+1} = \text{BAL}_t + d_t \cdot P_t, \text{ if the loan is modified at time } t.$$

- The modified loan is a *fixed-rate* mortgage with the maturity equal to the remainder of the initial loan, and the new modified interest rate, and thus the new monthly payment upon loan modification, is specified as a function of the initial monthly payment, initial interest rate, initial loan balance, margin rate, and states of the property. We estimate this process for the modified monthly payment directly from the data and by the year of the mortgage origination.

### 3.4 Payoff Function

We specify a borrower's current-period payoff from taking action $j$ in period $t$ as

$$u_j(x_t) + \epsilon_{jt},$$

where $u_j(x_t)$ is a deterministic function of $x_t$ and $\epsilon_{jt}$ is a choice-specific preference shock. The vector $\epsilon_t \equiv (\epsilon_{1t}, \cdots \epsilon_{J(x_t)t})$ is drawn from Type-I Extreme Value distribution, and we assume that $\epsilon_t$ is independently and identically distributed over time.

When a borrower with $d$ late payments makes $n$ monthly payments but does not pay off the mortgage, we assume that the deterministic part of his period-$t$ payoff is:

$$u_n(x_t) = \begin{cases} \beta_1 P_t + \beta_2 (n-1) P_t + \beta_3 \text{CS}_t + \beta_4 P_t \times \text{CS}_t + \beta_5 (n-1) P_t \times \text{CS}_t & \text{if } n \geq 1 \\ +\beta_6 Y_0 + \beta_7 \Delta\text{UNR}_t + \beta_8 X_0 + \xi_d + \zeta_n, & \\ \xi_d, & \text{if } n = 0. \end{cases} \tag{4}$$

The first term $\beta_1 P_t$ represents the disutility from one month's payment. The second term $\beta_2(n-1)P_t$ is the disutility of $(n-1)$-months' payment.[21] The term $\beta_3 \text{CS}_t$ determines the borrower's ability (or willingness) to make a payment. Specifically, $\text{CS}_t$ is the borrower's updated current credit score provided by TransUnion, and it captures not only the borrower's past payment history but also his ability to obtain future credit. We also allow credit scores to interact with borrowers' payment decisions, $P_t$ and $(n-1)P_t$, and the parameters $\beta_4$ and $\beta_5$ capture those interaction effects. The term $Y_0$ represents the borrower's income at origination, and $\Delta\text{UNR}_t$ captures the deviations of the current local market condition relative to its long-run average. The term $X_0$ is a collection of the borrower's characteristics at origination, which contains original monthly payment amount $(P_0)$, inverse loan-to-value ratio at origination $(ILTV_0)$, the year of loan origination, and whether the borrower's income is fully documented. $\xi_d$ is a dummy variable for the borrower's payment status

[20] As shown in Table 3, a small percentage of modified loans (about 5 percent) received a balance reduction in our sample. We assume that these borrowers are "surprised" by the unexpected changes in their loan balance. In our future research, we will endogenously determine the lenders' choices of new mortgage balances and interest rates upon modification.

[21] We use $\beta_1 P_t + \beta_2(n-1)P_t$, instead of a single term $\beta_1 n P_t$ to allow for the possibility that paying more than a single monthly payment amount could have a different utility cost than making only one payment.

$d$ at the beginning of the period. In order to reduce the number parameters to be estimated, we assume that for $d \geq 4$,

$$\xi_d = \xi_{4,0} + d\xi_{4,1}$$

Finally, $\zeta_n$ is a constant for taking action $n$. We also make the assumption that for $n \geq 4$,

$$\zeta_n = \zeta_{4,0} + n\zeta_{4,1}.$$

We normalize $\zeta_0 = 0$ because only relative utility is identified in a discrete choice model.

When a borrower chooses to pay off the mortgage ($j$ = PO), the deterministic part of the flow payoff is:

$$u_{PO}(x_t) = \beta_9 \sum_{t'=t+1}^{T} \delta^{t'} + \beta_{10} PPN_t + \beta_{11} \mathrm{CS}_t + \beta_{12} Y_t + \beta_{13} ILTV_0 + \beta_{14} ILTV_t + \zeta_{PO,d}, \qquad (5)$$

Where $\delta$ is the discount factor (which we set to be 0.99 in our estimation), $PPN_t$ is an indicator for whether the borrower has to pay a prepayment penalty if prepaying in period $t$; $ILTV_t$ is the ratio of the borrower's current housing price to the remaining balance (i.e., the inverse of mortgage loan-to-value ratio); and $ILTV_0$ is the inverse mortgage loan-to-value ratio at origination.[22] We assume that the model is terminated when the borrower pays off the mortgage.[23] $\zeta_{PO,d}$ determines the utility from paying off depending on the borrower's payment status $d$ at the beginning of the period. As before, in order to reduce the number parameters to be estimated, we assume that for $d \geq 3$,

$$\zeta_{PO,d} = \zeta_{PO,3,0} + d\zeta_{PO,3,1}.$$

If the house is liquidated, then as we mentioned earlier, the borrower's continuation value is given by:

$$V_t(\text{LIQUIDATED}) = \zeta_{liquid,state}. \qquad (6)$$

Note that we allow $\zeta_{liquid,state}$ to depend on the state of the property in order to capture state-level differences that are not captured by the model, such as legislative differences regarding the foreclosure process. We normalize $\zeta_{liquid,NV}$ to zero.

If the borrower does not pay off the mortgage by period $T$, and if the borrower's house is not liquidated by period $T$, the borrower reaches the final period $T$.[24] The model is then terminated,

[22] We assume that the house price follows an AR(1) process with the shock drawn from a normal distribution. The inverse of a normal random variable, however, does not have mean. In the analysis, we therefore use the inverse loan-to-value ratio $ILTV$ instead of the mortgage loan-to-value ratio.

[23] We make this assumption because the mortgage loan exits our database once the borrower pays off or refinances the mortgage.

[24] To simplify the problem, we do not follow mortgages to their actual terminal period, that is, 360 months. As shown in the data section, most borrowers either pay off their mortgages or become seriously delinquent within the first six years after mortgage origination.

and the borrower receives the terminal payoff:

$$V_T(x_T) = \begin{cases} \beta_{15} + \beta_{16} CS_T + \beta_{17} ILTV_T, & \text{if current at } T \\ 0, & \text{otherwise.} \end{cases} \tag{7}$$

**Remark 2.** *In our framework, we assume that the lender can directly affect a borrower's current-period flow utility only if the lender forecloses (i.e., liquidates) the house. If the lender chooses to modify the loan terms or wait, the borrower's flow utility is affected only to the extent that the modified loan term affects the borrower's monthly payment. Dynamically, the lender's choices obviously affect the borrower's ability to stay current in the mortgage and subsequently the probability of being foreclosed.*

### 3.5 Value Function

The borrower sequentially maximizes the sum of expected discounted flow payoffs in each period $t = 1, ..., T$. Let $\sigma_t(x_t, \epsilon_t)$ be the borrower's choice at time $t$ given the state vector $x_t$ and the vector of choice-specific shocks $\epsilon_t$, such that $\sigma_{t,j}(x_t, \epsilon_t) = 1$ if a borrower chooses action $j$ given $(x_t, \epsilon_t)$; and 0 otherwise. Let $\sigma \equiv (\sigma_1, ..., \sigma_T)$ denote the borrower's decision profile from period 1 to $T$ where $\sigma_T$, the terminal-period decision rule, is included for ease of exposition, but the borrower makes no choices (see the discussion prior to eq. (7)). We can then express the borrower's value functions from decision profile $\sigma \equiv (\sigma_1, ..., \sigma_T)$ recursively as follows: for $t \leq T - 1$,

$$V_t(x_t; \sigma) = \mathrm{E}_{\epsilon_t} \left[ \sum_{j \in J(x_t)} \sigma_{t,j}(x_t, \epsilon_t) \left\{ u_j(x_t) + \epsilon_{jt} + \delta \int_{x_{t+1} \in X_t} V_{t+1}(x_{t+1}; \sigma) dF(x_{t+1} | x_t, j) \right\} \right], \tag{8}$$

and $V_T(x_T; \sigma)$ is given by (7). The borrower's optimal decision rule $\sigma^*$ is such that $V_t(x_t; \sigma^*) \geq V_t(x_t; \sigma)$ for any possible decision rule $\sigma$, and for all $x_t$, where $t = 1, \cdots, T$.

## 4 Estimation

We define the choice-specific value function for action $j$ in period $t \leq T - 1$, $v_{t,j}(x_t)$, under decision profile $\sigma^*$, as

$$v_{t,j}(x_t) = u_j(x_t) + \delta \int_{x_{t+1} \in X_t} V_{t+1}(x_{t+1}; \sigma^*) dF(x_{t+1} | x_t, j). \tag{9}$$

The value function $V_t(x_t; \sigma^*)$ can then be written as:

$$V_t(x_t; \sigma^*) = \mathrm{E}_{\epsilon_t} \left[ \sum_{j \in J(x_t)} \sigma^*_{t,j}(x_t, \epsilon_t) \{ v_{t,j}(x_t) + \epsilon_{jt} \} \right]. \tag{10}$$

In order to solve for the optimal decision profile $\sigma^*$, we use backward induction following the standard methods in dynamic discrete choice models with finite periods (see, for example, Rust

(1987, 1994a, and 1994b) and Keane and Wolpin (1997)). We start from the penultimate period $T-1$. The choice-specific value function in period $T-1$ is given by:

$$v_{T-1,j}(x_{T-1}) = u_j(x_{T-1}) + \delta \int_{x_T \in X_T} V_T(x_T; \sigma^*) dF(x_T | x_{T-1}, j),$$

where $V_T(x_T; \sigma^*)$ is given by (7), and $\sigma_T^*$ is null. The optimal decision rule in period $T-1$ is then:

$$\sigma^*_{T-1,j}(x_{T-1}, \epsilon_{T-1}) = 1 \text{ iff } v_{T-1,j}(x_{T-1}) + \epsilon_{j,T-1} \geq \max_{j' \in J(x_{T-1})} \left\{ v_{T-1,j'}(x_{T-1}) + \epsilon_{j',T-1} \right\}. \tag{11}$$

Given the functional-form assumption for $\epsilon_{T-1}$, we can show, following Rust (1987), that

$$V_{T-1}(x_{T-1}; \sigma^*) = \ln \left( \sum_{j' \in J(x_{T-1})} \exp(v_{T-1,j'}(x_{T-1})) \right) + \gamma, \tag{12}$$

where $\gamma$ is Euler's constant.

Now let us consider the borrower's optimal decision rule in period $T-2$. In order to calculate $v_{T-2,j}(x_{T-2})$, we need to know $\int_{x_{T-1} \in X_{T-1}} V_{T-1}(x_{T-1}; \sigma^*) dF(x_{T-1} | x_{T-2}, j)$, which can be calculated using equation (12) and the state transition function $F\left(x_{T-1} | x_{T-2}, j\right)$. We then derive $\sigma^*_{T-2,j}(x_{T-2}, \epsilon_{T-2})$ and $V_{T-2}(x_{T-2}; \sigma^*)$ analogous to what we did in period $T-1$. We repeat this process until we reach the initial period. The borrower's optimal decision rule in period $t$ is:

$$\sigma^*_{t,j}(x_t, \epsilon_t) = 1 \text{ if } v_{t,j}(x_t) + \epsilon_{jt} \geq \max_{j' \in J(x_t)} \left\{ v_{t,j'}(x_t) + \epsilon_{j't} \right\}, \tag{13}$$

and the period-$t$ continuation value function is:

$$V_t(x_t; \sigma^*) = \ln \left( \sum_{j' \in J(x_t)} \exp\left(v_{t,j'}(x_t)\right) \right) + \gamma. \tag{14}$$

Moreover, a borrower's conditional choice probability under the optimal decision profile $\sigma^*$ for alternative $j \in J(x_t)$ in period $t$ when the state vector is $x_t$ is given by:

$$p_{t,j}(x_t; \sigma^*) = \mathrm{E}_{\epsilon_t}[\sigma^*_{t,j}(x_t, \epsilon_t)] = \frac{\exp(v_{t,j}(x_t))}{\sum_{j' \in J(x_t)} \exp(v_{t,j'}(x_t))}. \tag{15}$$

We estimate the model using maximum likelihood. In the data, we observe a path of states and choices for each individual $i$: $(x^i, a^i) \equiv \{(x_t^i, a_t^i)\}_{t=1}^T$, where $a_t^i \equiv \{a_{jt}^i\}_{j \in J(x_t^i)}$, and $a_{jt}^i$ is defined to be a dummy variable that equals one when individual $i$ chooses action $j$ in period $t$. The likelihood of observing $(x^i, a^i)$ given initial state $x_1^i$ and parameter vector $\theta$ for individual $i$ is:

$$L(x^i, a^i | x_1^i; \theta) = \prod_{t=1}^{T-1} l_t(a_t^i, x_{t+1}^i | x_t^i; \theta), \tag{16}$$

where $\prod_{t=1}^{T-1} l(a_t^i, x_{t+1}^i | x_t^i; \theta)$ is the likelihood of observing action $a_t^i$ in period $t$ and observing the state to transition to $x_{t+1}^i$ in period $t+1$ given state $x_t^i$ and parameter vector $\theta$, as predicted by the model, and it is given by:

$$l_t(a_t^i, x_{t+1}^i | x_{it}; \theta) = \prod_{j \in J(x_t^i)} \left[ p_{t,j}(x_t^i; \sigma^*(\theta)) f(x_{t+1}^i | x_t^i, j) \right]^{a_{jt}^i}, \tag{17}$$

where $p_{t,j}(\cdot; \cdot)$ is given by (15) and $\sigma^*(\theta)$ is the model's predicted optimal decision profile for the borrower given parameter vector $\theta$. Parameter estimate $\theta^*$ maximizes the log-likelihood for the whole sample, that is,

$$\begin{aligned} \theta^* &= \arg\max \ln L(\theta) = \sum_{i=1}^{I} \ln \left( L(x^i, a^i | x_1^i; \theta) \right) \\ &= \arg\max \sum_{i=1}^{I} \sum_{t=1}^{T-1} \sum_{j \in J(x_t^i)} a_{jt}^i \left[ \ln \left( p_{t,j}(x_t^i; \sigma^*(\theta)) \right) + \ln f(x_{t+1}^i | x_t^i, j) \right]. \end{aligned} \tag{18}$$

# 5 Estimation Results

## 5.1 Lender's Decisions

As previously discussed, we estimate the lender's policy functions parametrically using logit or multinomial logit regressions. The borrower enters period $t$ with a delinquent status $d_t$ and makes the payment decision $a_t$, after which the lender makes the decisions regarding whether to modify, liquidate, or do nothing about the loan based on the delinquent status of the loan at the end of the period $t$.[25] However, in the data we only observe the loan status at the beginning of the period. Thus when we observe that a loan was current in period $t$ and was also modified in period $t$, we assume that the loan would have been one month late at the end of period $t$, had the modification not taken place.

Specifically, we estimate lender's decisions separately for four categories of loans:

**Category 1:** $(d_t = 0, a_t = 0)$. Borrowers are *current* in the beginning of the period but do not make payments in the period;

**Category 2:** $(d_t = 1, a_t = 0)$. Borrowers are *one month delinquent* in the beginning of the period but do not make payments in the period;

**Category 3:** $(d_t = 2, a_t = 0)$. Borrowers are *two months delinquent* in the beginning of the period but do not make payments in the period;

**Category 4:** $(d_t \geq 3, a_t = 0)$. Borrowers are *three or more months delinquent* at the beginning of a period but do not make payments in the period.

It is important to note that the lender only modifies or liquidates a loan if the borrower does *not* make any payment in the period. Therefore, if a borrower enters the period with loan status $d_t \geq 1$,

[25] We do not separately model the lender's decisions regarding when to start foreclosure. As long as foreclosure is not complete, we consider the lender as "waiting."

and if he makes $a_t \geq 1$ payment, the lender's only choice is to wait even though the status of the loan at the end of the period may still be one or more months delinquent if $a_t < d_t + 1$.

In our specification of the lender's decisions, we recall from Table 2 that a lender *almost never* liquidates a house whose mortgage is less than three months delinquent. Thus, we assume that for loans in categories 1 to 3, the lenders choose only between modification and waiting; the probability of modification is specified as a logit function of the state variables that includes borrower characteristics and loan status.[26] For loans in category 4, we assume that the lender considers three options: modification, liquidation, and waiting. We specify a multinomial logit function to represent the lender's probabilities of choosing the three alternatives. We further condition the lender's decisions on state and year of origination. Finally, we also estimate the lender's decisions on interest rates for modified loans. Given the much smaller number of modified loans, we only condition this decision on mortgage year of origination. In total, we have 51 regressions (4 states $\times$ 3 origination years $\times$ 4 loan status + 3 origination years for interest rate estimation). To save space, we only report the estimation results for lenders' modification, foreclosure, or wait decisions for loans originated in 2006 in Florida in Appendix Tables A4 and A5. Estimation results for interest rates after modification for loans originated in year 2006 are reported in Table A6.[27]

***Category 1 Loans.*** For category 1 loans originated in Florida in 2006, lenders are more likely to modify if borrowers have high credit scores, high monthly payments but low initial monthly payments, and full documentation. Older loans are also more likely to be modified. By contrast, mortgage loans with high initial mortgage loan-to-value ratios and three-year fixed-interest periods are less likely to be modified.

***Category 2 Loans.*** For category 2 loans originated in Florida in 2006, the factors that explain modification probability are similar to those that are current at the beginning of the period with a few exceptions. Income at origination reduces the probability of being modified, while increases in local unemployment rates relative to recent trends raise the modification probability.

***Category 3 Loans.*** For category 3 loans originated in Florida in 2006, similar factors determine the likelihood of loans being modified by lenders as those for Category 2 loans. The only exception is that loan-to-value ratio at origination and credit scores no longer matter for modification probability.

***Category 4 Loans.*** For category 4 loans, we include many more explanatory variables to our multinomial logit regressions. Loans are more likely to be modified if income at origination is low, loan-to-value ratio is low, initial loan-to-value ratio is high, loans are older, or they have full documentation. Loans, however, are less likely to be modified if borrowers have many missed

[26]In our estimation, we dropped the few (specifically, four cases) loans of category 1 to 3 that were liquidated; that is, we assume that the four borrowers were making choices assuming that foreclosure would not have happened yet. We did not include their terminal liquidation in the likelihood function to avoid degeneracy.

[27]To increase the precision, we use the full sample, instead of the 1.75 percent random sample, in estimating lenders' decisions.

payments. Given the number of missing payments, high loan-to-value ratio increases the probability of modification. Most modifications occur when loans are between five and nine months delinquent.

In terms of liquidation, current credit score, income at origination, mortgage loan-to-value ratio, and months of delinquency all increase the probability significantly. Current monthly payment, loan age, and full documentation all reduce the probability of liquidation. Given the number of missing payments, a higher mortgage loan-to-value ratio reduces the liquidation probability. Finally, most liquidations occur when loans are 11 or 12 months behind in payments.

### 5.2 New Interest Rate and Monthly Payment Following Modification

As indicated in Table A6, for loans originated in 2006, the new interest rate increases with the interest rate at origination, margin rate, mortgage balance at origination, income at origination, mortgage loan-to-value ratio, and whether the loan has full documentation, but the new interest rate decreases with current credit score, remaining balance, loan-to-value ratio at origination, loan age, deviation of local unemployment rates from recent trends, and number of months that the borrower is behind in payments. Of the four states, everything else equal, Florida has the lowest modified loan rates.

### 5.3 Estimates of the Stochastic Processes

In Section 3.2, we described that borrowers and lenders have beliefs about the stochastic processes that govern the evolution of LIBOR rates, the local housing prices, local unemployment rates, and credit scores. We assume that the borrowers have rational expectations about these processes and estimate them using the *ex post* realizations of these processes. The estimates are reported in Table 4. Note that the process of log credit score is endogenous for the borrower because its evolution depends on the payment status of the mortgage loan, whose evolution depends on the borrower's payment decisions.

As can be seen, all the variables depend strongly on their lagged values (i.e., they exhibit strong persistence). For credit scores, missing mortgage payments also have a significantly negative impact on their values.

### 5.4 Borrower's Payoff Function Parameters

Table 5 presents the coefficient estimates in the three payoff functions associated with the three payment decisions. From Panel A, we observe that a borrower overall derives negative flow utility from making more payments; moreover, his flow utility from making a single payment is higher when his credit score is higher, but the flow utility from making more than one payment is lower if he has a higher credit score. His flow utility from making a payment is lower when the local unemployment rate is high relative to its recent historical average. In terms of conditions at origination, a borrower's flow utility from making payments improves with his initial income and the initial amount of the payment. High house value relative to mortgages (or low mortgage loan-to-value ratio) at origination and full documentation increase the propensity to make payments.

| Coefficient | Estimate | Standard Errors |
|---|---|---|
| Panel A: $\text{Libor}$: $\ln(\text{Libor}_{t+1}) = \lambda_0 + \lambda_1 \ln(\text{Libor}_t) + \epsilon_{\text{Libor},t}$ | | |
| $\lambda_0$ | -0.013 | (0.010) |
| $\lambda_1$ | 0.996*** | (0.009) |
| $\sigma_{\text{Libor}}$ | 0.09656*** | (0.00106) |
| Panel B: House Price $h_{t+1} = \lambda_2 + \lambda_3 h_t + \epsilon_{h,t}$ | | |
| $\lambda_2$ | 0.671*** | (0.010) |
| $\lambda_3$ | 0.997*** | (0.000) |
| $\sigma_h$ | 2.5419*** | (0.00979) |
| Panel C: Local Unemp. Rates $\Delta\text{Unr}_{t+1} = \lambda_4 + \lambda_5 \Delta\text{Unr}_t + \epsilon_{\text{Unr},t}$ | | |
| $\lambda_4$ | 0.049*** | (0.007) |
| $\lambda_5$ | 0.959*** | (0.003) |
| $\sigma_{\text{Unr}}$ | 0.90066*** | (0.00979) |
| Panel D: Credit Score: | | $\ln(\text{CS}_{t+1}) = \lambda_6 + \lambda_7 \ln(\text{CS}_t) + \lambda_8 1[d=1]$ $+\lambda_9 1[d=2] + \lambda_{10} 1[d=3] + \lambda_{11} 1[d \geq 4] + \epsilon_{\text{CS},t}$ |
| $\lambda_6$ | 0.149*** | (0.001) |
| $\lambda_7$ | 0.897*** | (0.001) |
| $\lambda_8$ | -0.072*** | (0.001) |
| $\lambda_9$ | -0.164*** | (0.002) |
| $\lambda_{10}$ | -0.130*** | (0.002) |
| $\lambda_{11}$ | -0.007*** | (0.000) |
| $\sigma_{\text{CS}}$ | 0.17719*** | (7.93e-05) |

Table 4: Coefficient Estimates for Stochastic Processes

Notes: ***, **, and * denote statistical significance at 1%, 5%, and 10%, respectively.

Turning to the constants associated with each payment status at the beginning of the period captured by $\xi_0$ to $\xi_{4+}$, the model requires relatively larger values associated with more months delinquent in order to explain the payment rate for such borrowers. For constants associated with payment decisions captured by $\zeta_0$ to $\zeta_{4+}$, the high disutility the borrower suffers from making large numbers of payments indicates his reluctance (or inability) to do so.

From Panel B, we see that the borrower's repayment decisions are negatively correlated with prepayment penalty. A borrower with higher current credit score, high initial income, high current house value relative to mortgage, and low house value relative to mortgage at origination is more likely to pay off his mortgage. The more payments that the borrower has missed, the less likely he will be able to pay off his mortgage by either refinancing or selling the house.

From Panel C, we see that if the house is liquidated, the payoffs to the borrower are lower in California and Florida than in Nevada. Finally, from Panel D, we see that the borrower's payoff function at the terminal period $T$ is not well identified as none of the variables are significant.

## 6 Model Fit

In order to gauge the fit of our model, we present figures that compare the model's predictions for the distributions of endogenous variables with empirical analogs in the data. Figure 1 compares the probabilities of missing payments and prepayment conditional on the delinquency status at the beginning of the period in the data with those predicted by our estimated model. The model does

| Coefficient | Estimate | Std. Err. |
|---|---|---|
| Panel A: Coefficients in $u_n(x_t)$ as specified in (4) | | |
| $P_t : (\beta_1)$ | -0.1285*** | (0.0117) |
| $(n-1)P_t : (\beta_2)$ | 0.2689** | (0.0062) |
| $CS_t : (\beta_3)$ | 0.0866*** | (0.0040) |
| $P_t \times CS_t : (\beta_4)$ | 0.0003 | (0.0021) |
| $(n-1)P_t \times CS_t : (\beta_5)$ | -0.1077*** | (0.0021) |
| $Y_0 : (\beta_6)$ | 0.0209** | (0.0052) |
| $\Delta UNR_t : (\beta_7)$ | -0.0130*** | (0.0013) |
| $P_0 : (\beta_{8,1})$ | 0.1154*** | (0.0096) |
| $ILTV_0 : (\beta_{8,2})$ | 0.0153** | (0.0072) |
| Full Doc: $(\beta_{8,3})$ | 0.0033** | (0.0017) |
| Constant: $(\xi_0)$ | -0.4961*** | (0.0496) |
| Constant: $(\xi_1)$ | -1.3017*** | (0.0468) |
| Constant: $(\xi_2)$ | -1.3880*** | (0.0481) |
| Constant: $(\xi_3)$ | -1.6704*** | (0.3436) |
| Constant: $(\xi_{4,0})$ | 0.5403*** | (0.0518) |
| Constant: $(\xi_{4,1})$ | -0.0143*** | (0.0033) |
| Constant: $(\zeta_1)$ | 0.0761 | (0.0491) |
| Constant: $(\zeta_2)$ | -2.1488*** | (0.0922) |
| Constant: $(\zeta_3)$ | -3.0994*** | (0.1417) |
| Constant: $(\zeta_{4,0})$ | 0.6790* | (0.3540) |
| Constant: $(\zeta_{4,1})$ | -0.5983*** | (0.0318) |
| Panel B: Coefficients in $u_{PO}(x_t)$ as specified in (5) | | |
| $\sum_{t'=t+1}^{T} \delta^{t'} : (\beta_9)$ | 0.1012*** | (0.0081) |
| $PPN_t : (\beta_{10})$ | -2.6110*** | (0.0765) |
| $CS_t : (\beta_{11})$ | 0.6368*** | (0.0136) |
| $Y_0 : (\beta_{12})$ | 0.3302** | (0.1521) |
| $ILTV_t : (\beta_{13})$ | 8.5867*** | (0.1708) |
| $ILTV_0 : (\beta_{14})$ | -6.3451*** | (0.2791) |
| Constant: $(\zeta_{PO,0})$ | -7.2565*** | (0.5160) |
| Constant: $(\zeta_{PO,1})$ | -8.3419*** | (0.5138) |
| Constant: $(\zeta_{PO,2})$ | -8.1471*** | (0.5182) |
| Constant: $(\zeta_{PO,3,0})$ | -5.7989*** | (0.5209) |
| Constant: $(\zeta_{PO,3,1})$ | -0.3685*** | (0.0310) |
| Panel C: Coefficients in $V_t(\text{LIQUIDATED})$ as specified in (6) | | |
| $\zeta_{liquid,AZ}$ | 0.3358 | 0.2363 |
| $\zeta_{liquid,CA}$ | -1.0123*** | 0.2008 |
| $\zeta_{liquid,FL}$ | -3.5621*** | 0.2688 |
| Panel D: Coefficients in $V_T(x_T)$ as specified in (7) | | |
| Constant $(\beta_{15})$ | -3.3825 | (65.9577) |
| $CS_t$ $(\beta_{16})$ | -0.0847 | (4.0928) |
| $ILTV_T$ $(\beta_{17})$ | -0.5666 | (43.7830) |

Table 5: Coefficient Estimates for Borrowers' Payoff Functions

Notes: ***, **, and * denote statistical significance at 1%, 5%, and 10%, respectively.

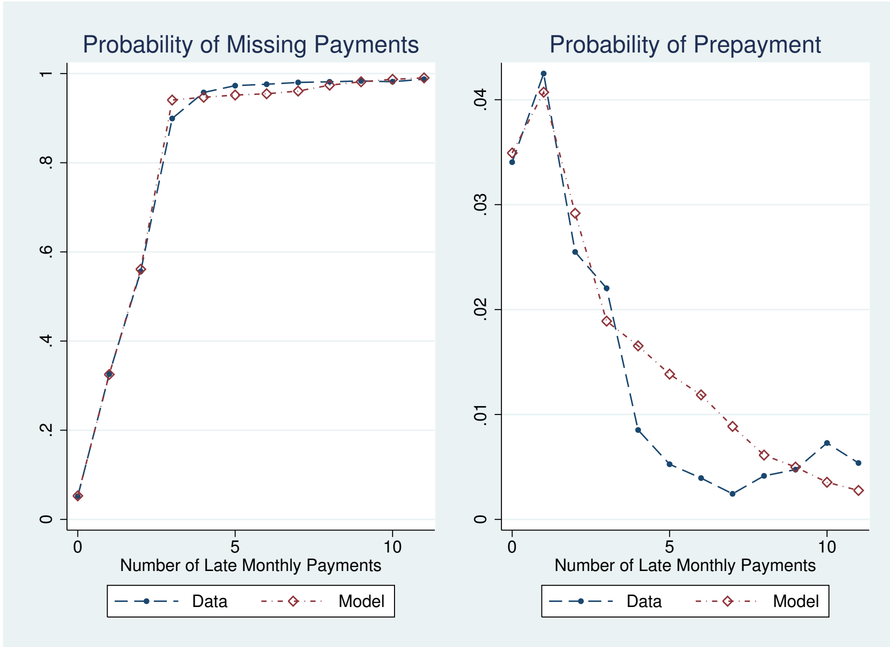


Figure 1: Probabilities of Missing Payments and Prepayment by Beginning-of-Period Delinquency Status

a good job of capturing the patterns in the data. The more payments a borrower misses, the more likely that he will miss payments again; more important, once the borrower is three months or more behind on his payment schedule, he will almost certainly stay delinquent. The model also captures the relationship between months of delinquency and the probability of prepayment; interestingly, the model predicts that the probability of prepayment is highest among borrowers who are one month late with their payments.

Figure 2 compares the probabilities of missing payments and prepayment by loan age in the data with those predicted by our model. Note that while we capture the probability of default by loan age well, the match with the probability of prepayment is less than perfect, partly because the data are more volatile. Both curves are hump-shaped with the probability of default or staying default peaking at age 36 months, roughly one year after the majority of the loans exited their fixed-teaser-rate period. The peak of prepayment, by contrast, occurs at 24 months, the time when the majority of the loans' fixed-teaser-rate periods expire.

Figure 3 charts the probabilities of missing payments and prepayment by the ratio of current monthly mortgage payment to initial monthly payment (when the loan was originated). The fits are good for both charts. Interestingly, there is a large jump of about 50 percentage points in default probability when the current payment exceeds the initial payment, consistent with the observations we documented earlier that a borrower has a higher probability of default shortly after his mortgage payment resets to a higher value. After that, the probability of default declines somewhat and then hovers at around 50 percent. The prepayment probability, on the other hand, increases consistently with the increase in the current mortgage payment relative to the initial mortgage payment after the initial drop following the reset in interest rates. Because a loan leaves our sample after it is

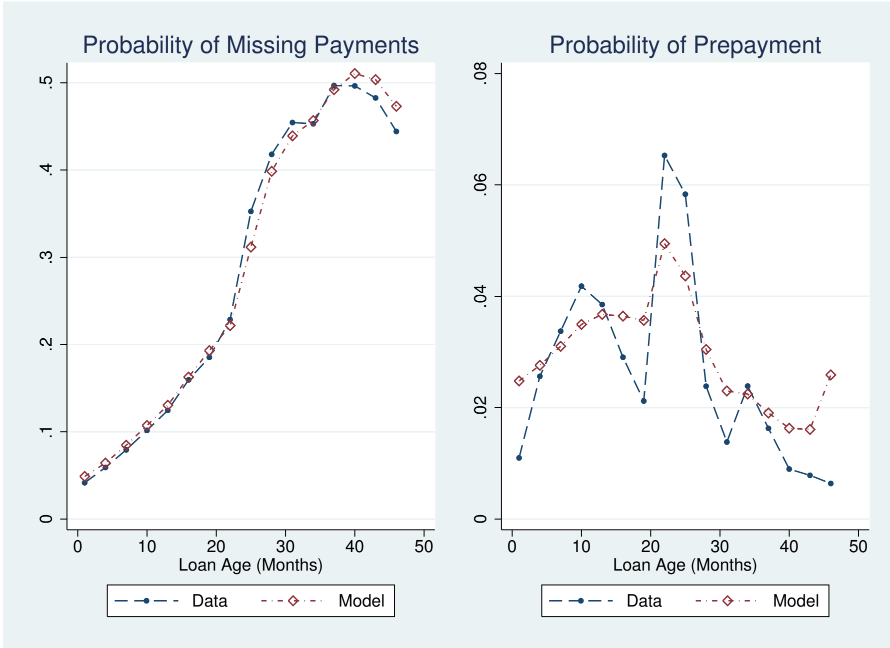


Figure 2: Probabilities of Missing Payments and Prepaymen by Loan Age
Note: We group the loans into age intervals in months, 1-3, 4-6, ..., 43-45, 46+, in the calculation of the probabilities.

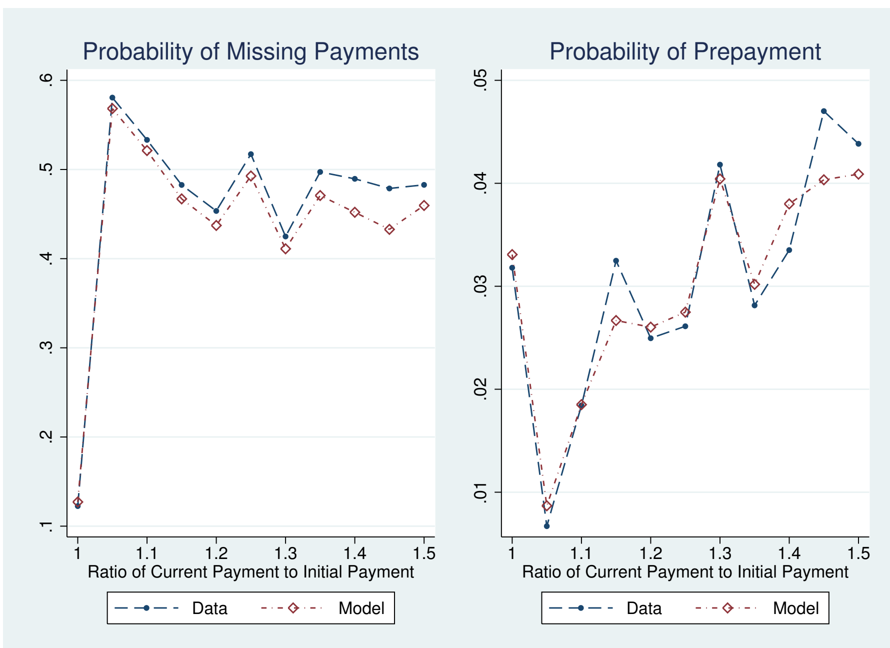


Figure 3: Probabilities of Missing Payments and Prepayment by Relative Monthly Payment Ratio.
Notes: (1) Relative monthly payment is the ratio of current monthly payment to the initial monthly payment when the loan was originated. (2) We group loans into intervals of relative payment ratio, 1-1.05, 1.05-1.1, 1.1-1.15, ..., 1.45-1.50, in the calculation of the probabilities.

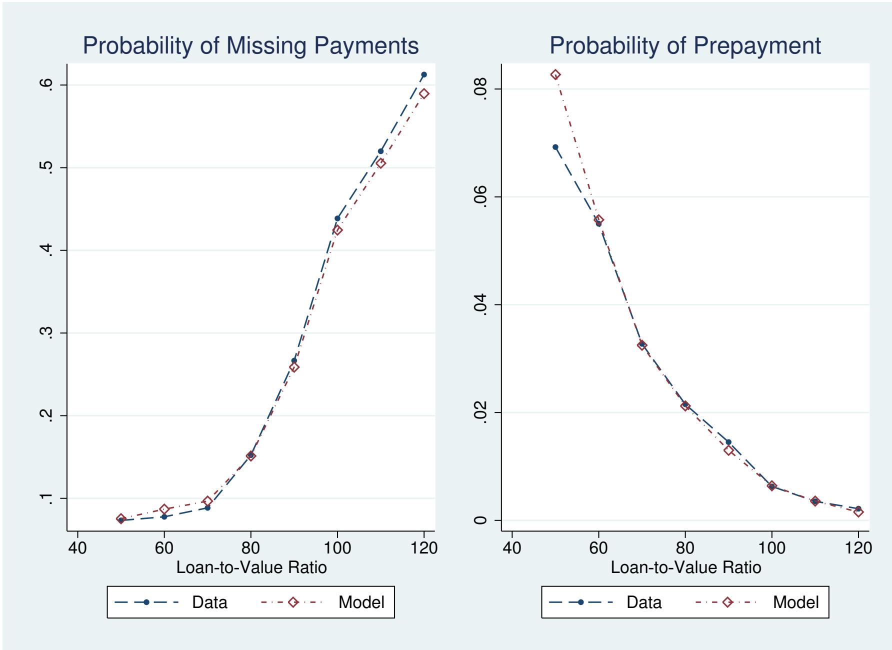


Figure 4: Probabilities of Missing Payments and Prepayment by the Current Mortgage Loan-to-Value (LTV) Ratio.
Notes: (1) Unit for LTV is in percentage. (2) We group loans into intervals of LTVs, 50-, [50, 60), [60,70), ..., [110,120), 120+, in the calculation of the probabilities.

prepaid, the default pattern depicted in the figure cannot be interpreted as direct evidence that interest rate reset necessarily leads to higher default rate as pointed out in Fuster and Willen (2015). We will address this issue in detail in the next section.

Figure 4 depicts the probabilities of missing payments and prepayment by the current mortgage loan-to-value ratio. The model does a good job at capturing the patterns in both series. As expected, the higher the current mortgage loan-to-value ratio is, the more likely the borrower will default and the less likely he will prepay.

Finally, Figure 5 charts the probabilities of missing payments and prepayment by the borrowers' current credit scores. The model captures the default probability better than it captures the prepayment probability. Note that credit scores capture the borrowers' past payment histories as well as future payment ability. Not surprisingly, the higher the credit scores are, the less likely the borrowers will default. In other words, borrowers with higher credit scores are more likely to make mortgage payments on time and are also more likely to prepay.

# 7 Counterfactual Simulations

In this section, we report counterfactual simulation results to address two sets of questions. The first set of simulations is aimed at a quantitative understanding of the roles of different factors that contributed to subprime borrowers' default and prepayment behavior during the housing crisis. The second set of simulations is aimed at the policies, particularly monetary policies and alternative mortgage designs, that may help reduce defaults.

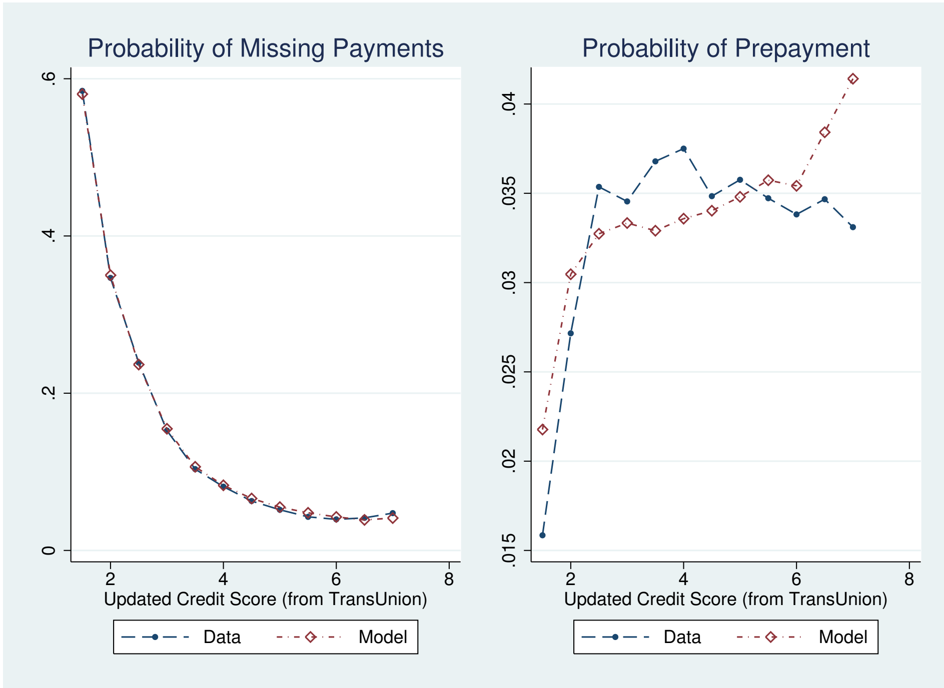


Figure 5: Probabilities of Missing Payments and Prepayment by Credit Score
Notes: (1) Credit score units are by 100. (2) We group loans into intervals of credit scores, 150-, [150, 200), ..., [650,700), 700+, in the calculation of the probabilities.

It is useful to start out with some basic facts about the changes in monthly payments, housing prices, and unemployment rates that the ARM borrowers in our data set face as their loans age. In Figure 6, we show the average monthly payment amounts as loans age, for 2/28 (2 years fixed rate, 28 years adjustable rate) and 3/27 (3 years fixed rate, 27 years adjustable rate) ARMs. It shows that upon the end of the initial lower teaser rate period, borrowers' monthly payments would typically increase substantially for loans that originated in 2004 and 2005; in contrast, it would decrease substantially for loans that originated in 2006. These observations are not surprising as interest rates lowered substantially after 2007.

In Figure 7, we plot the percentage changes of local housing prices and local unemployment rates by loan age for loans that were originated in 2004, 2005, and 2006, respectively. It shows that for loans that were originated in 2004, the local housing prices experienced on average more than 30 percent gains before declining around the time these loans reached about 24 months of loan age; for loans that were originated in 2005, there was also a modest (about 10 percent) housing price gain up to a loan age of 12 months before the housing market crash. In contrast, the loans that were originated in 2006 immediately experienced housing price declines as deep as 45 percent. Similarly, the experience of the loans in terms of labor market conditions as measured by local unemployment rates also differs substantially by loan origination years. Loans originated in later years faced much tougher labor market conditions marked by high unemployment rates. The differences by loan origination year on these dimensions explain why the effects of a variety of counterfactual changes differ by loan origination years, as we discuss next.

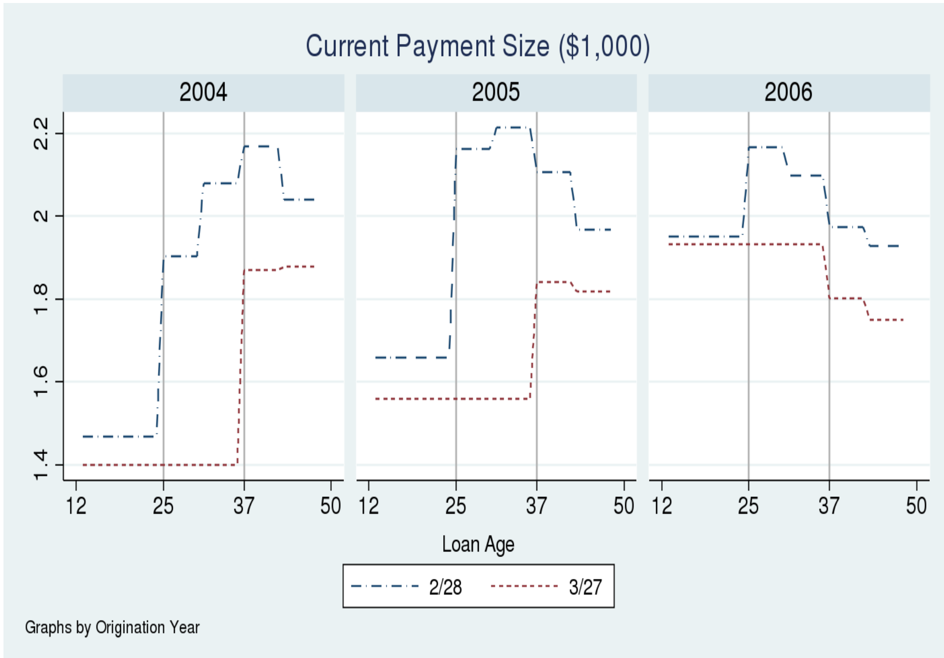


Figure 6: Current Monthly Payment Transition by Loan Age and ARM Type


Figure 7: Housing Price and Unemployment Rate Trends by Year of Origination of Loans

| Loan | Baseline | | | | Fixed-Rate Mortgage | | | |
|---|---|---|---|---|---|---|---|---|
| Age | Current | Paid off | Delinquent | [Liquidated] | Current | Paid off | Delinquent | [Liquidated] |
| Panel A: Loans Originated in 2004 | | | | | | | | |
| 18 | 0.360 | 0.584 | 0.056 | 0.011 | 0.388 | 0.564 | 0.048 | 0.010 |
| 24 | 0.220 | 0.735 | 0.045 | 0.016 | 0.253 | 0.698 | 0.049 | 0.014 |
| 30 | 0.127 | 0.821 | 0.052 | 0.020 | 0.166 | 0.787 | 0.048 | 0.017 |
| 36 | 0.075 | 0.871 | 0.054 | 0.025 | 0.115 | 0.838 | 0.047 | 0.020 |
| 42 | 0.044 | 0.896 | 0.060 | 0.030 | 0.075 | 0.868 | 0.057 | 0.024 |
| Panel B: Loans Originated in 2005 | | | | | | | | |
| 18 | 0.444 | 0.457 | 0.099 | 0.015 | 0.498 | 0.405 | 0.098 | 0.014 |
| 24 | 0.319 | 0.564 | 0.117 | 0.029 | 0.400 | 0.493 | 0.107 | 0.025 |
| 30 | 0.220 | 0.636 | 0.144 | 0.045 | 0.300 | 0.565 | 0.135 | 0.036 |
| 36 | 0.144 | 0.671 | 0.185 | 0.063 | 0.217 | 0.604 | 0.179 | 0.050 |
| 42 | 0.094 | 0.689 | 0.217 | 0.082 | 0.140 | 0.628 | 0.232 | 0.073 |
| Panel C: Loans Originated in 2006 | | | | | | | | |
| 18 | 0.460 | 0.315 | 0.226 | 0.031 | 0.490 | 0.284 | 0.226 | 0.025 |
| 24 | 0.315 | 0.351 | 0.334 | 0.063 | 0.372 | 0.322 | 0.306 | 0.055 |
| 30 | 0.216 | 0.376 | 0.408 | 0.105 | 0.265 | 0.352 | 0.383 | 0.097 |
| 36 | 0.156 | 0.393 | 0.451 | 0.156 | 0.190 | 0.374 | 0.436 | 0.142 |
| 42 | 0.122 | 0.404 | 0.474 | 0.216 | 0.158 | 0.387 | 0.455 | 0.195 |

Table 6: Role of Interest Rate Reset

Notes: (1) The "Baseline" columns report the model's prediction of the loan status under the *actual* loans; the "Fixed-Rate Mortgage" columns report the counterfactual results when all ARMs were converted to fixed-rate mortgages with interest rates fixed at the initial teaser rates of their corresponding ARMs. (2) The numbers reported in the table are the fractions of loans in different statuses, "Current," "Paid Off," or "Delinquent." The loans that are "Liquidated" are also included in "Delinquent" status. The total fractions in "Current," "Paid Off," and "Delinquent" statuses add up to 1.

### 7.1 Understanding the Factors for Defaults and Prepayments

**Interest Rate Resets.** The amount of the monthly mortgage payment in an ARM is fixed for a few (typically, two to three) years initially and then resets every six months. The initial fixed rate is, in general, lower than the rate after the interest rate resets. Because of an increase in mortgage payments upon the reset, many commentators believed that the massive number of defaults by subprime mortgage borrowers in the recent financial crisis was attributable to the reset of ARM interest rates. To quantify how much the initial reset of ARMs contributed to the subprime borrowers' default and prepayment rates observed in the data, we simulate the model under the counterfactual scenario that the interest rate is *fixed* at the initial teaser rate for the entire duration of the loan.

In Table 6, we report the model's predictions regarding the percentage of loans in different statuses (Current, Paid Off, Delinquent, or Liquidated) at different loan ages, for loans originated in 2004, 2005, and 2006, respectively. The columns under "Baseline" are the model's prediction of the loan status under the *actual* loans,[28] and the columns under "Fixed-Rate Mortgage" are the model's prediction of the loan status if all ARMs were replaced by fixed-rate mortgages with interest rates fixed at the initial teaser rates of the ARMs.

[28] Below we will repeatedly compare our counterfactual results with the results in the "Baseline" of Table 6.

Under the baseline, the left side of Table 6 shows that the performance of the loans differs substantially depending on the year of origination. Loans originated in 2004 are much more likely to be paid off over time than become delinquent or be liquidated. By 36 months of loan age when the initial interest rate resets occurred, 87.1 percent of the loans were already paid off (i.e., refinanced or prepaid by selling); 5.4 percent of the loans were in various stages of delinquency, including 2.5 percent being liquidated. The performance of the loans that originated in 2005 were quite different. By 36 months of loan age, 67 percent of these loans were paid off, and 18.5 percent were in different stages of delinquency, including 6.3 percent in foreclosure. The loans originated in 2006 faced even more difficulty, as 45.1 percent were in delinquency, including an astonishingly high 15.6 percent in foreclosure at 36 months of loan age. These differential outcomes of loans that originated in different years are the result of many factors, including the dynamics of the interest rates, local unemployment rates, and local housing prices, as depicted in Figures 6 and 7.

The right side of Table 6 presents the performance of the loans if all the ARMs were converted to fixed-rate mortgages at the initial teaser rates. It shows that, in general, changing the ARMs to fixed-rate mortgages alone, thus taking away the interest rate resets of the ARMs, has a very limited effect on the delinquency and liquidation rates. For loans originated in 2004, the delinquency rate at 36 months of loan age would be 4.7 percent under the fixed-rate mortgages instead of the 5.4 percent under the original ARMs; similarly, at 36 months of age the delinquency rates would be 17.9 percent and 43.6 percent for loans originated in 2005 and 2006, in contrast to 18.5 percent and 45.1 percent respectively under the original ARMs.[29] The margin on which the fixed-rate mortgages seem to have a bigger effect is the "Current" and "Paid Off" margin; for example, the fraction of current loans at 36 months of loan age would be 11.5 percent (7.5 percent), 21.7 percent (14.4 percent), and 19.0 percent (15.6 percent) respectively for loans originated in 2004, 2005, and 2006, under the fixed-rate mortgages (respectively, under the original ARMs).

**Declining Housing Prices.** Many researchers have argued that negative home equity is important in a borrower's default decision (see, e.g., Bhutta, Dokko, and Shan, 2010; and Foote, Gerardi and Willen, 2008 and 2012). In Table 7, we report counterfactual simulation results to understand the role of substantial housing price declines for the loans we study.

We conduct two counterfactual experiments. In the first, we ask what would have happened to the delinquency and foreclosure rates had the housing prices remained *unchanged* from the origination of the mortgages, i.e., $\widetilde{h}_t = h_0$ for all $t \geq 1$. In the second, we explore the interaction of interest rate resets of the ARMs and local housing market conditions by assuming in addition that all the ARMs are converted to fixed-rate mortgages with interest rates fixed at the initial teaser rates (as on the right side of Table 6).

On the left side, we report the results from the first counterfactual experiment, $\widetilde{h}_t = h_0$ for all $t \geq 1$. As should be expected from Figure 7, setting housing prices unchanged at levels of mortgage origination would have deprived the substantial housing price gains for loans originated in 2004 and,

[29]It is important to point out that our calculation of loan status is over all loans including those that are paid off. By doing so, we avoid the selection bias issue raised in Fuster and Willen (2015), where they argue that when less-risky loans were refinanced, the delinquency rates of remaining loans would be by definition higher.

| Loan | $\widetilde{h}_t = h_0$ | | | | $\widetilde{h}_t = h_0$ & Fixed-Rate Mortgage | | | |
|---|---|---|---|---|---|---|---|---|
| Age | Current | Paid off | Delinquent | [Liquidated] | Current | Paid off | Delinquent | [Liquidated] |
| Panel A: Loans Originated in 2004 | | | | | | | | |
| 18 | 0.530 | 0.324 | 0.146 | 0.038 | 0.499 | 0.349 | 0.152 | 0.040 |
| 24 | 0.445 | 0.398 | 0.157 | 0.063 | 0.416 | 0.425 | 0.159 | 0.069 |
| 30 | 0.322 | 0.495 | 0.183 | 0.083 | 0.335 | 0.489 | 0.176 | 0.088 |
| 36 | 0.223 | 0.550 | 0.227 | 0.107 | 0.257 | 0.536 | 0.207 | 0.104 |
| 42 | 0.140 | 0.589 | 0.271 | 0.132 | 0.175 | 0.572 | 0.253 | 0.129 |
| Panel B: Loans Originated in 2005 | | | | | | | | |
| 18 | 0.531 | 0.358 | 0.112 | 0.021 | 0.522 | 0.380 | 0.098 | 0.020 |
| 24 | 0.436 | 0.450 | 0.114 | 0.034 | 0.429 | 0.465 | 0.106 | 0.031 |
| 30 | 0.284 | 0.570 | 0.146 | 0.046 | 0.325 | 0.553 | 0.121 | 0.042 |
| 36 | 0.182 | 0.642 | 0.176 | 0.062 | 0.233 | 0.612 | 0.155 | 0.055 |
| 42 | 0.109 | 0.679 | 0.212 | 0.079 | 0.152 | 0.652 | 0.196 | 0.069 |
| Panel C: Loans Originated in 2006 | | | | | | | | |
| 18 | 0.459 | 0.471 | 0.070 | 0.012 | 0.449 | 0.487 | 0.063 | 0.007 |
| 24 | 0.341 | 0.594 | 0.066 | 0.017 | 0.342 | 0.608 | 0.050 | 0.010 |
| 30 | 0.195 | 0.735 | 0.071 | 0.021 | 0.211 | 0.738 | 0.051 | 0.013 |
| 36 | 0.122 | 0.801 | 0.077 | 0.025 | 0.133 | 0.809 | 0.058 | 0.018 |
| 42 | 0.088 | 0.835 | 0.077 | 0.033 | 0.079 | 0.859 | 0.062 | 0.027 |

Table 7: Role of Housing Prices and the Interaction with the Interest Rate Resets

Notes: (1) On the left side, we assume that the housing prices remained unchanged from those at loan origination; on the right side, we assume in addition that all the ARMs were converted to fixed-rate mortgages with interest rates fixed at the initial teaser rates of the corresponding ARMs. (2) The numbers reported in the table are the percentages of loans in different statuses: "Current," "Paid Off," or "Delinquent." The loans that are "Liquidated" are also included in "Delinquent" status. The total percentages in "Current," "Paid Off," and "Delinquent" statuses add up to 1.

to some extent, for the loans originated in 2005. Indeed, our counterfactual experiments show that our model predicted much higher (respectively, slightly higher) delinquency rates and foreclosure rates for 2004 loans (respectively, for 2005 loans) than in the baseline (see the left panel in Table 6). Had the housing prices stayed constant at the loan origination, the delinquency and liquidation rates would be 22.7 percent and 17.6 percent at 36 months of loan age for loans originated in 2004 and 2005, in contrast to 5.4 percent and 18.5 percent, respectively, under the baseline. The liquidation rates would be 10.7 percent instead of 2.5 percent for loans originated in 2004 at 36 months of loan age; for loans originated in 2005, the liquidation rates would be higher than the baseline if the housing prices stayed at $h_0$, but they would be slightly lower than those in the baseline after 24 months. This precisely reflects the fact that, for loans originated in 2005, housing prices actually started to fall below the level at the loan origination at around the loan age of 23 months (see the left side of Figure 7).

In striking contrast, from Figure 7 we know that the 2006 loans experienced housing price declines immediately in the data; thus, setting the housing prices as unchanged at their origination levels would lead to much lower delinquency and foreclosure rates. Indeed, our counterfactual results for the 2006 loans confirm these: Had housing prices not declined so precipitously, our model predicts that the delinquency rates for loans originated in 2006 would be about 7 percent at all loan ages, and the cumulative liquidation rates would reach 3.3 percent at loan age of 42 months.

| Loan Age | $\Delta\text{Unr}_t = \Delta\text{Unr}_0$ | | | |
|---|---|---|---|---|
| | Current | Paid off | Delinquent | [Liquidated] |
| Panel A: Loans Originated in 2004 | | | | |
| 18 | 0.409 | 0.536 | 0.054 | 0.008 |
| 24 | 0.278 | 0.676 | 0.046 | 0.012 |
| 30 | 0.162 | 0.793 | 0.045 | 0.015 |
| 36 | 0.100 | 0.854 | 0.046 | 0.020 |
| 42 | 0.056 | 0.890 | 0.055 | 0.025 |
| Panel B: Loans Originated in 2005 | | | | |
| 18 | 0.524 | 0.388 | 0.088 | 0.013 |
| 24 | 0.437 | 0.468 | 0.095 | 0.022 |
| 30 | 0.294 | 0.565 | 0.141 | 0.032 |
| 36 | 0.192 | 0.615 | 0.193 | 0.052 |
| 42 | 0.125 | 0.642 | 0.233 | 0.079 |
| Panel C: Loans Originated in 2006 | | | | |
| 18 | 0.549 | 0.260 | 0.191 | 0.029 |
| 24 | 0.454 | 0.297 | 0.249 | 0.056 |
| 30 | 0.337 | 0.330 | 0.333 | 0.098 |
| 36 | 0.269 | 0.353 | 0.378 | 0.149 |
| 42 | 0.222 | 0.371 | 0.407 | 0.201 |

Table 8: Role of Labor Market Conditions

Note: The numbers reported in the table are the percentages of loans in different statuses: "Current," "Paid Off," or "Delinquent." The loans that are "Liquidated" are also included in "Delinquent" status. The total percentages in "Current," "Paid Off," and "Delinquent" statuses add up to 1.

On the right side, we see that adding the assumption that the interest rates would be fixed at the initial teaser rates of the ARMs for the total duration of the loan only generates a rather small effect. Comparing with the results on the left side, we find that making the mortgages fixed-rate rather than adjustable-rate makes the loans more likely to be paid off for young loans originated in 2004 and 2005 and all loans originated in 2006. Delinquency rates are slightly lower for loans originated in 2005 and 2006 at all ages, and for older loans originated in 2004. For loans originated in 2004, the delinquency rates do not change much when the loans are young.

**Labor Market Conditions.** In Table 8, we simulate the role of local unemployment rates on the observed borrowers' delinquency and foreclosure decisions. Gerardi, Herkenhoff and Ohanian (2013) document that individual unemployment is a strong predictor of default using data from the Panel Study of Income Dynamics. We assume that the local unemployment rate stayed the same as that at loan origination. The results show that because of their increased payment ability, borrowers are more likely to stay current with their mortgage payment, less likely to pay off, and less likely to default. These effects are stronger for loans originated in 2006. The reason is that for loans that originated in 2004 and 2005, the local unemployment rates did not increase initially. In contrast, local unemployment rates increased almost immediately after origination for loans originated in 2006 as shown in Figure 7.

| Loan | $\text{LIBOR}_t = 0$ | | | |
|---|---|---|---|---|
| Age | Current | Paid off | Delinquent | [Liquidated] |
| Panel A: Loans Originated in 2004 | | | | |
| 18 | 0.383 | 0.560 | 0.057 | 0.009 |
| 24 | 0.263 | 0.689 | 0.048 | 0.015 |
| 30 | 0.160 | 0.794 | 0.046 | 0.019 |
| 36 | 0.104 | 0.846 | 0.050 | 0.023 |
| 42 | 0.062 | 0.881 | 0.057 | 0.027 |
| Panel B: Loans Originated in 2005 | | | | |
| 18 | 0.492 | 0.417 | 0.092 | 0.013 |
| 24 | 0.391 | 0.504 | 0.105 | 0.025 |
| 30 | 0.270 | 0.585 | 0.145 | 0.037 |
| 36 | 0.183 | 0.627 | 0.191 | 0.053 |
| 42 | 0.119 | 0.647 | 0.234 | 0.075 |
| Panel C: Loans Originated in 2006 | | | | |
| 18 | 0.483 | 0.287 | 0.230 | 0.029 |
| 24 | 0.368 | 0.324 | 0.308 | 0.055 |
| 30 | 0.241 | 0.355 | 0.403 | 0.096 |
| 36 | 0.175 | 0.372 | 0.453 | 0.146 |
| 42 | 0.134 | 0.387 | 0.479 | 0.204 |

Table 9: Impacts of Traditional Monetary Policy: Setting the LIBOR Rate to Zero

Note: The numbers reported in the table are the percentages of loans in different statuses: "Current," "Paid Off," or "Delinquent." The loans that are "Liquidated" are also included in "Delinquent" status. The total percentages in "Current," "Paid Off," and "Delinquent" statuses add up to 1.

## 7.2 Potential Policy Responses to Reduce Defaults

In this subsection, we evaluate the effectiveness of several potential policy responses to reduce default and foreclosure rates. We first consider the role of monetary policy and then consider the role of alternative mortgage contract designs.

### 7.2.1 Traditional Monetary Policy

There are recent works that looked at how ARM borrowers responded to a decrease in their mortgage interest rates due to a low short-term interest rate (LIBOR). General findings in the works are that monetary policy can have positive effects on ARM borrowers because their interest rates are tied to short-term interest rates. In particular, ARM borrowers are less likely to default (Fuster and Willen 2015) and more likely to increase consumption due to a larger disposable income (Keys et al. 2014, and Di Maggio, Kermani, and Ramcharan 2014).

In Table 9, we report the counterfactual results from an experiment where LIBOR rate is set to zero, and, as a result, the ARM borrowers' monthly payment amounts will be determined by the lifetime floor interest rate once the teaser rate period of the ARM expires. This could provide the best case scenario (or upper bound) on how much monetary policy may reduce the delinquency and foreclosure rates.

It is important to point out that setting the LIBOR rate to zero does not necessarily imply that the borrowers' monthly payments will be that much lower than their payments in the teaser periods. The reason is that, as we mentioned in Section 3.2, the majority of the ARMs have lifetime

| Year of Origination | 2004 | 2005 | 2006 |
|---|---|---|---|
| Teaser Rate (%) | 6.78 | 7.03 | 8.00 |
| Margin (%) | 5.63 | 5.71 | 5.92 |
| LIBOR at Initial Reset (%) | 5.30 | 4.88 | 2.66 |
| Margin + LIBOR (%) | 10.93 | 10.59 | 8.58 |
| Lifetime Floor Rate (%) | 6.39 | 6.58 | 7.49 |

Table 10: Average Loan Characteristics by Year of Origination

floor rates, which would be applied even when the LIBOR rate is zero. In fact, most borrowers' monthly payments would only decrease slightly when the LIBOR rate is zero upon the reset of the interest rate. Also note that as reported in Table 10, in the data margin rates and lifetime floor rates were high and LIBOR rates were already low for 2006 loans; thus setting the LIBOR rate to zero had little effect on loans originated in 2006. Therefore, the results in Table 9 suggests that setting the LIBOR rate at zero would reduce the mortgage pay off rates of almost all loans; however, mortgage delinquency rates and liquidation rates change *little.*

#### 7.2.2 Automatic Loan Modification Contingent on Housing Price Index with Cushions

If a housing price downturn leads to massive default rates, then one way to mitigate this problem is to link the mortgage monthly payment to the current housing price index. Caplin et al. (2007), Shiller (2008a, 2008b), Posner and Zingales (2009), Mian and Sufi (2014), and Kung (2015) have suggested that such "continuous workout mortgages" might have reduced the mortgage default and foreclosure. Piskorski and Tchistyi (2010, 2011) show that the optimal mortgage contracts in the presence of stochastic housing price appreciation or uncertain income and uncertain mortgage rates all have some forms of loss sharing between borrowers and lenders such as balance or interest rate reduction when housing prices decline or when income decreases or interest rates jump. In their model, interest rates are exogenous and the optimal plan involves complex home equity lines. We consider two different automatic loan modification schemes in this subsection.[30]

**Modification of Monthly Payments Only.** We first consider the case in which only the monthly payment amount is automatically modified as housing prices change. Specifically, denote $\tilde{P}_t$ as the modified monthly payment at period $t$, and $P_t$ as the monthly payment amount in the absence of modification according to the original loan. Let $h_t$ and $h_0$ denote the housing price index at period $t$ and at origination, respectively. The first counterfactual we consider assumes that the monthly payment will be automatically modified from $P_t$ to $\tilde{P}_t$ as follows:

$$\tilde{P}_t = P_t \times \min\left\{1, \kappa \times \frac{h_t}{h_0}\right\}, \text{ where } \kappa \geq 1, \tag{19}$$

[30]Kung (2015) studies the general equilibrium effect of "continuous workout mortgage" on housing prices and mortgage interest rates. Borrowers in his model are only allowed to make the current monthly payment or to refinance (i.e., delinquency and foreclosure are not the focus of his paper).

while the *principal balance is not adjusted.*[31] In equation (19), the parameter $\kappa \geq 1$ can be used to adjust how much *cushion* is afforded to the seller in terms of housing price declines before the automatic modification of monthly payments (and loan balance below) is activated. We refer to $\kappa$ as the "*cushion parameter*": The higher $\kappa$ is, the more housing price decline is required to trigger the automatic modification. For example, as we will experiment below, when $\kappa = 1.15$, the housing price would have to decline by 13 percent ($\approx 1 - 1/\kappa$) from that at loan origination before monthly payments are reduced.

**Modification of Principal Balance (and Monthly Payments, Too)** In the second counterfactual, we assume that

$$\widetilde{BAL}_t = BAL_t \times \min\left\{1, \kappa \times \frac{h_t}{h_0}\right\}, \tag{20}$$

where as in (19), $\kappa \geq 1$ is the cushion parameter. Because monthly payment is proportional to principal balance, as we showed in (2), the automatic modification of principal balance will also automatically adjust the monthly payment.[32]

A key feature of the automatic modification mortgages of both forms (19) and (20) is that modifications are triggered by *the housing price declines alone*, not at all by the delinquency status of the borrowers, which are subject to potential strategic behavior by borrowers. This feature distinguishes the automatic modification from the stochastic loan modification of the form as modeled in Section 3.3.

In this subsection, we are also interested in evaluating the impact of alternative mortgage contracts on the revenue of the lenders. For this purpose, we make the following assumption:[33]

**Assumption 1.** *Upon foreclosure, the lender receives 75 percent of the house value.*

It is worth making three observations. First, the baseline corresponds to the case of $\kappa = +\infty$ (i.e., the monthly payment actually never deviates from those in the baseline). Second, everything else equal, as long as $\kappa \geq 1$, the borrower is always made better off under both (19) and (20) than under the baseline in expectation. The closer $\kappa$ is to 1, the easier that housing price decline triggers reduction in monthly payments under (19), or both loan balance and monthly payments under (20), and thus the better off the borrowers are.

Third, changes in the cushion parameter $\kappa$ in the automatic modification mortgages (19) and (20) have two effects. First, different values of $\kappa$ affect the borrowers' payment behavior, resulting in different levels of delinquency and liquidation. Intuitively, and as we will show below, the closer $\kappa$ is to 1, the smaller the fraction of mortgages that are liquidated and thus the smaller the social surplus destruction in the 25 percent loss of the house value in foreclosure (see Assumption 1). Second, $\kappa$ affects how the surplus from the reduction in foreclosure is shared between the borrowers and the lender. When $\kappa$ is closer to 1, while it is true that the automatic modification

[31] Stochastic loan modification of the form as modeled in Section 3.3 stays as in the baseline in the counterfactual experiments in this subsection.

[32] This is akin to "partially shared appreciation mortgage" considered in Kung (2015).

[33] Campbell, Giglio, and Pathak (2011) find that the average discount of a house value for foreclosures is about 27 percent.

mortgages (19) and (20) prevent more foreclosure, the lenders receive smaller shares of the surplus because the foreclosure reduction is achieved solely by the lenders reducing monthly payments and/or principals. At higher values of the cushion parameter $\kappa$, borrowers will also share some of the "sacrifices" for reduction in foreclosure as smaller declines in housing prices would not trigger the automatic reduction. As a result, while the borrowers' ex ante expected value under automatic modification mortgages is always higher than that in the baseline, and decreasing in $\kappa$, the lender's expected revenue is non-monotonic in $\kappa$. In particular, to the extent that it is important that lenders *voluntarily* adopt the automatic modification mortgages of the form (19) or (20), we would like to explore whether there are values of $\kappa$ that *both* the borrowers and lenders are better off than the baseline. Otherwise, we would expect that lenders would have to raise interest rates to compensate for their revenue loss.

**Automatic Modification Mortgages Without Cushions:** $\kappa = 1$. In Table 11, we present the results from counterfactual simulations under automatic modification mortgages, without cushions (i.e., when $\kappa = 1$). We should emphasize that this version of automatic modification corresponds to most of what has been studied in the literature.[34]

On the left side, we present the results under the automatic modification mortgages (19) that adjust payment sizes only, with $\kappa = 1$. We find that the automatic modification mortgages only slightly reduce the delinquency and foreclosure rates for loans originated in 2004 and 2005; however, the delinquency and foreclosure rates are significantly reduced for loans originated in 2006. The delinquency rate for 2006 loans under the "just payment size" automatic modification mortgages is reduced from 45.1 percent under the baseline to 35.0 percent at 36 months of loan age; the liquidation rate is lowered from 15.6 percent to 11.1 percent at 36 months of loan age and from 21.6 percent to 15.7 percent at 42 months of loan age. As we mentioned earlier, it is important to know how such automatic modification mortgages might affect lenders' revenues. We also calculate lenders' revenue as the present value of the borrowers' expected payments. If borrowers prepay, their payments in that period will just be the remaining mortgage balances. If borrowers' houses are liquidated, we assume that the lender receives 75 percent of the estimated current house value as stated in Assumption 1. As stated in the notes for Table 11, the lender's revenues per borrower under the baseline are \$221.7K, \$230.5K, and \$216.3K, respectively, for loans originated in 2004, 2005, and 2006. Comparing these numbers with those obtained in Table 11, we see that the automatic modification loans of the form (19) lowers the lender's revenue per borrower, unfortunately, for loans originated in 2005 and 2006. In particular, the per-borrower revenue for 2005 loans is \$228.8K, about \$1,700 (or about 0.74 percent) less per borrower than under the baseline, and the per-borrower revenue for 2006 loans is \$212.5K, about \$3,800 (or about 1.76 percent) less per borrower under the baseline.

On the right side, we present the results under the automatic modification mortgages (20) that adjust the loan balance and thus also monthly payment, with $\kappa = 1$. We find that automatic modification mortgages that lower loan balance still have little effect on the outcomes of loans originated in 2004 relative to the baseline; however, the impacts on loans originated in 2005 and

[34] For example, this is the case considered in Kung's (2015) "partially shared appreciation mortgage."

| Loan | Just Payment Size ($\kappa = 1$) | | | | Payment Size and Balance ($\kappa = 1$) | | | |
|---|---|---|---|---|---|---|---|---|
| Age | Current | Paid off | Delinquent | [Liquidated] | Current | Paid off | Delinquent | [Liquidated] |
| | | | | Panel A: Loans Originated in 2004 | | | | |
| 18 | 0.385 | 0.563 | 0.051 | 0.008 | 0.379 | 0.570 | 0.051 | 0.008 |
| 24 | 0.235 | 0.720 | 0.045 | 0.011 | 0.236 | 0.723 | 0.041 | 0.011 |
| 30 | 0.137 | 0.819 | 0.044 | 0.015 | 0.136 | 0.820 | 0.044 | 0.014 |
| 36 | 0.082 | 0.869 | 0.049 | 0.020 | 0.080 | 0.872 | 0.049 | 0.019 |
| 42 | 0.051 | 0.898 | 0.052 | 0.025 | 0.045 | 0.902 | 0.052 | 0.024 |
| Revenue | | | 221.7K | | | | 221.7K | |
| | | | | Panel B: Loans Originated in 2005 | | | | |
| 18 | 0.447 | 0.458 | 0.095 | 0.014 | 0.443 | 0.468 | 0.089 | 0.014 |
| 24 | 0.328 | 0.562 | 0.110 | 0.025 | 0.316 | 0.586 | 0.098 | 0.024 |
| 30 | 0.234 | 0.632 | 0.134 | 0.038 | 0.211 | 0.673 | 0.117 | 0.033 |
| 36 | 0.164 | 0.667 | 0.169 | 0.053 | 0.143 | 0.721 | 0.136 | 0.045 |
| 42 | 0.111 | 0.685 | 0.204 | 0.071 | 0.095 | 0.754 | 0.151 | 0.059 |
| Revenue | | | 228.8K | | | | 229.7K | |
| | | | | Panel C: Loans Originated in 2006 | | | | |
| 18 | 0.525 | 0.285 | 0.190 | 0.025 | 0.469 | 0.380 | 0.150 | 0.021 |
| 24 | 0.426 | 0.317 | 0.257 | 0.047 | 0.352 | 0.477 | 0.171 | 0.038 |
| 30 | 0.344 | 0.340 | 0.316 | 0.076 | 0.250 | 0.575 | 0.175 | 0.054 |
| 36 | 0.296 | 0.353 | 0.350 | 0.111 | 0.181 | 0.646 | 0.172 | 0.070 |
| 42 | 0.254 | 0.365 | 0.380 | 0.157 | 0.137 | 0.699 | 0.164 | 0.086 |
| Revenue | | | 212.5K | | | | 212.8K | |

Table 11: Automatic Modification of Payment Size and Principal Balance Without Cushions ($\kappa = 1$)

Notes: (1) The numbers reported in the table are the percentages of loans in different statuses: "Current," "Paid Off," or "Delinquent." The loans thare are "Liquidated" are also included in "Delinquent" status. The total percentages in "Current," "Paid Off," and "Delinquent" statuses add up to 1. (2) The numbers in the row labeled "Revenue" refer to lender's expected revenue per borrower, under Assumption 1. The lender's expected revenue per borrower under the baseline is $221.7K, $230.5K, and $216.3K, respectively, for loans originated in 2004, 2005, and 2006.

2006 are much bigger than those under the “Just Payment Size” automatic modification loans. For loans originated in 2005, automatic reductions in loan balances reduce the delinquency rate at the 36 months of loan age to 13.6 percent, in contrast to 18.5 percent under the baseline and 16.9 percent under the “Just Payment Size” automatic-modification mortgages. For loans originated in 2006, the delinquency rate is now 17.2 percent, in contrast to 45.1 percent under the baseline and 35.0 percent under the “Just Payment Size” automatic-modification mortgages. The reduction in foreclosure rate for loans originated in 2006 is also astonishing: At 36 months of loan age, it is 7.0 percent under the balance automatic-modification mortgages, in contrast to 11.1 percent under the “Just Payment Size” automatic-modification mortgages and 15.6 percent under the baseline. The reductions in delinquency and foreclosure are mostly achieved by increases in the fraction of paid off loans.

The lender’s expected revenues per borrower are also lower under this type of automatic modification mortgages than those under the baseline. For loans originated in 2005, the per-borrower revenue under the counterfactual mortgages is \$229.7K, about \$800 less than that under the baseline, and for 2006 loans, the per-borrower revenue is \$212.8K, about \$3,500 lower than that under the baseline. However, it is also interesting to note that the lender’s expected revenue per borrower is actually higher under the seemingly more generous automatic balance modification mortgages (20) than under the automatic modification mortgages (19) that only adjust the monthly payment. This is due to the fact that the more generous automatic balance modification mortgages are very successful in reducing the costly foreclosure, allowing the lenders to more than recoup the cost of the generosity in lowering the mortgage balances as well as monthly payments.

**Automatic Modification Mortgages with Cushions:** $\kappa = 1.15$. One issue with the auto-matic modification mortgages without cushions studied in Table 11 is that the lender’s revenue is lower than that in the baseline. In Table 12, we show that it is possible to adjust the cushion parameter $\kappa$ to $\kappa = 1.15$ so that the lender’s per-borrower revenue is *at least as high as* that in the baseline for loans generated in all years. This ensures that lenders are also better off under the proposed automatic modification mortgages with cushions. As is obvious qualitatively and as we will show quantitatively in Table 14, borrowers are better off than the baseline under the cushioned automatic modification mortgages as well; thus, this represents a Pareto improvement over the baseline mortgages.

The left side of Table 12 shows that at $\kappa = 1.15$, the automatic modification loans that only adjust the payment sizes have little impact on the borrower outcomes, obtaining only very slight reductions in delinquency and liquidation rates relative to the baseline (see the left side in Table 6) for loans originated in all years, though the lender’s revenue also increases slightly relative to the baseline. However, the right side shows that automatic modification mortgages that reduce the mortgage balances when the housing price declines by about 13 percent ($= 1 - 1/1.15$) are able to achieve moderate reductions in delinquency and liquidation rates for loans originated in 2005, and more sizable reductions for loans originated in 2006. Specifically, we find that under the automatic modification mortgages that reduce balances with $\kappa = 1.15$, for loans originated in 2005, the delinquency rate at 42 months of loan age decreases from 21.7 percent in the baseline to 19.3

| Loan | Just Payment Size ($\kappa = 1.15$) | | | | Payment Size and Balance ($\kappa = 1.15$) | | | |
|---|---|---|---|---|---|---|---|---|
| Age | Current | Paid off | Delinquent | [Liquidated] | Current | Paid off | Delinquent | [Liquidated] |
| | | | | Panel A: Loans Originated in 2004 | | | | |
| 18 | 0.378 | 0.569 | 0.053 | 0.008 | 0.366 | 0.581 | 0.053 | 0.011 |
| 24 | 0.229 | 0.725 | 0.046 | 0.013 | 0.226 | 0.728 | 0.045 | 0.014 |
| 30 | 0.136 | 0.819 | 0.045 | 0.017 | 0.134 | 0.819 | 0.047 | 0.018 |
| 36 | 0.082 | 0.868 | 0.051 | 0.021 | 0.081 | 0.867 | 0.052 | 0.022 |
| 42 | 0.049 | 0.895 | 0.055 | 0.027 | 0.046 | 0.897 | 0.057 | 0.028 |
| Revenue | | | 221.8K | | | | 222.2K | |
| | | | | Panel B: Loans Originated in 2005 | | | | |
| 18 | 0.436 | 0.466 | 0.097 | 0.013 | 0.434 | 0.468 | 0.098 | 0.014 |
| 24 | 0.312 | 0.572 | 0.116 | 0.024 | 0.317 | 0.571 | 0.112 | 0.026 |
| 30 | 0.214 | 0.642 | 0.144 | 0.040 | 0.216 | 0.647 | 0.137 | 0.039 |
| 36 | 0.148 | 0.675 | 0.177 | 0.057 | 0.150 | 0.684 | 0.165 | 0.054 |
| 42 | 0.097 | 0.695 | 0.209 | 0.077 | 0.098 | 0.710 | 0.193 | 0.073 |
| Revenue | | | 230.8K | | | | 230.7K | |
| | | | | Panel C: Loans Originated in 2006 | | | | |
| 18 | 0.477 | 0.308 | 0.215 | 0.027 | 0.469 | 0.322 | 0.209 | 0.029 |
| 24 | 0.364 | 0.339 | 0.297 | 0.059 | 0.360 | 0.378 | 0.262 | 0.060 |
| 30 | 0.280 | 0.367 | 0.353 | 0.095 | 0.265 | 0.444 | 0.291 | 0.090 |
| 36 | 0.228 | 0.381 | 0.391 | 0.134 | 0.203 | 0.495 | 0.302 | 0.122 |
| 42 | 0.193 | 0.393 | 0.414 | 0.184 | 0.164 | 0.533 | 0.302 | 0.154 |
| Revenue | | | 216.7K | | | | 216.3K | |

Table 12: Automatic Modification of Payment Size and Principal Balance with Cushions ($\kappa = 1.15$)

Notes: (1) The numbers reported in the table are the percentages of loans in different statuses: "Current," "Paid Off," or "Delinquent." The loans that are "Liquidated" are also included in "Delinquent" status. The total percentages in "Current," "Paid Off," and "Delinquent" statuses add up to 1. (2) The numbers in the row labeled "Revenue" refer to the lender's expected revenue per borrower, under Assumption 1. The lender's expected revenue per borrower under the baseline is \$221.7K, \$230.5K, and \$216.3K, respectively, for loans originated in 2004, 2005, and 2006.

percent, and the liquidation rate is reduced from 8.2 percent in the baseline to 7.3 percent; for loans originated in 2006, the delinquency rate at 43 months of loan age decreases from 47.4 percent in the baseline to 30.2 percent, and the liquidation rate is reduced from 21.6 percent in the baseline to 15.4 percent. At the same time, the lender's per-borrower revenue is at least as high as that in the baseline for all years. Even though the reductions in delinquency and liquidation rates under automatic-modification mortgages with a cushion parameter $\kappa = 1.15$ are not as large as those without cushions, it should be noted that the lenders would *not* have to increase their interest rate under the cushioned automatic-modification mortgages. It is also worth noting that borrowers are benefiting from the automatic modification mortgages regardless of whether they are eventually delinquent or liquidated, since one important feature of the automatic modification mortgages is that modifications are triggered by the housing price declines, not at all by the delinquency status of the borrowers, which are subject to potential strategic behavior by borrowers.

### 7.3 What If Lenders Can Commit *Not* to Modify Any Loans?

In this subsection, we consider a different counterfactual: What if lenders can commit not to modify any loans? This counterfactual can shed light on whether borrowers' strategic defaults, in order to receive loan modification, played any role in the observed delinquency and liquidation. As we will argue, this counterfactual will also shed light on why the percentage of loans that received modification (only 5.24 percent) during the housing crisis was so low.

To implement this counterfactual, we consider two scenarios that depend on whether the lender would replace each loan modification observed in the data by either the alternative of "wait and do nothing" or the alternative of "liquidation." The results are presented in Table 13.

On the left side, we assume that the lenders replace all modifications in the data by liquidation instead. We find that, with the additional threat of foreclosure, borrowers are much less likely to default on their mortgages than the baseline, and this effect is particularly strong for loans originated in 2005 and 2006; much higher percentage of loans are paid off. Because of the now higher probability of liquidation, although liquidation rates come down for all loans, the magnitude of the reductions of liquidation rates is much lower. Surprisingly, the lender's per-borrower revenues under this counterfactual policy are, respectively, \$224.6K, \$240.4K, and \$233.2K for loans originated in 2004, 2005, and 2006, which are, respectively, 1.3 percent, 4.3 percent, and 7.8 percent higher than their counterparts under the baseline. The increases in lender revenue are mainly due to more borrowers paying off their loans (despite prepayment penalty for most of the loans) and less delinquency.

On the right side, we assume that the lenders replace all modifications in the data by "waiting." We find that mortgage delinquency rates are also lower than the baseline, though the reduction is less pronounced than in the previous case where lenders replace modification by liquidation. As in the previous case, borrowers pay off more often than in the baseline when there is no prospect of loan modification. Note that the liquidation rate under this counterfactual policy is actually higher than that in the baseline for loans originated in 2006; after all, waiting and doing nothing by the lender in place of modification will still increase borrowers' months in delinquency, eventually

| Loan | No Modification, More Liquidation | | | | No Modifcation, More Waiting | | | |
|---|---|---|---|---|---|---|---|---|
| Age | Current | Paid off | Delinquent | [Liquidated] | Current | Paid off | Delinquent | [Liquidated] |
| Panel A: Loans Originated in 2004 | | | | | | | | |
| 18 | 0.336 | 0.630 | 0.034 | 0.008 | 0.340 | 0.625 | 0.035 | 0.009 |
| 24 | 0.199 | 0.774 | 0.027 | 0.012 | 0.206 | 0.767 | 0.027 | 0.011 |
| 30 | 0.111 | 0.860 | 0.029 | 0.015 | 0.119 | 0.853 | 0.028 | 0.014 |
| 36 | 0.064 | 0.905 | 0.031 | 0.019 | 0.068 | 0.898 | 0.033 | 0.018 |
| 42 | 0.035 | 0.930 | 0.036 | 0.024 | 0.039 | 0.924 | 0.037 | 0.025 |
| Revenue | 224.6K | | | | 224.3K | | | |
| Panel B: Loans Originated in 2005 | | | | | | | | |
| 18 | 0.390 | 0.562 | 0.048 | 0.012 | 0.408 | 0.540 | 0.051 | 0.012 |
| 24 | 0.270 | 0.675 | 0.055 | 0.019 | 0.291 | 0.652 | 0.057 | 0.020 |
| 30 | 0.183 | 0.750 | 0.066 | 0.031 | 0.202 | 0.725 | 0.073 | 0.030 |
| 36 | 0.127 | 0.784 | 0.089 | 0.044 | 0.139 | 0.760 | 0.102 | 0.047 |
| 42 | 0.079 | 0.801 | 0.120 | 0.062 | 0.084 | 0.776 | 0.140 | 0.065 |
| Revenue | 240.4K | | | | 238.8K | | | |
| Panel C: Loans Originated in 2006 | | | | | | | | |
| 18 | 0.433 | 0.439 | 0.128 | 0.031 | 0.455 | 0.416 | 0.129 | 0.034 |
| 24 | 0.323 | 0.496 | 0.182 | 0.063 | 0.335 | 0.462 | 0.203 | 0.064 |
| 30 | 0.235 | 0.529 | 0.235 | 0.101 | 0.221 | 0.497 | 0.283 | 0.107 |
| 36 | 0.167 | 0.548 | 0.285 | 0.140 | 0.147 | 0.518 | 0.336 | 0.159 |
| 42 | 0.120 | 0.560 | 0.320 | 0.189 | 0.106 | 0.530 | 0.364 | 0.217 |
| Revenue | 233.2K | | | | 230.0K | | | |

Table 13: Replacing Modification by Liquidation (Left Side) or by Waiting (Right Side)

Notes: (1) The numbers reported in the table are the percentages of loans in different statuses: “Current,” “Paid Off,” or “Delinquent.” The loans that are “Liquidated” are also included in “Delinquent” status. The total percentages in “Current,” “Paid Off,” and “Delinquent” statuses add up to 1. (2) The numbers in the row labeled “Revenue” refer to lender’s expected revenue per borrower, under Assumption 1. The lender’s expected revenue per borrower under the baseline is $221.7K, $230.5K, and $216.3K, respectively, for loans originated in 2004, 2005, and 2006.

increasing the probability of foreclosure. It may seem somewhat surprising that borrowers will be less likely to default when the lender commits not to modify any loans. The reason is actually quite simple: The ex ante value of being delinquent is much lower without any possibilities to get their loans modified, and, as the number of months in delinquency increases, the houses will eventually be foreclosed. So the value of being delinquent decreases substantially without modification. In other words, the presence of modification possibility led some borrowers to delinquency in the hopes of getting their interest rates reduced.

Again, it is interesting to note that the lenders' expected per-borrower revenue under this counterfactual policy is $224.4K, $238.8K and $230.0K respectively for loans originated in 2004, 2005, and 2006, which are respectively 1.2 percent, 3.6 percent, and 6.3 percent higher than their counterparts under the baseline.

**Discussion.** The counterfactual results reported in Table 13 suggest that the lenders would have been able to raise their expected revenues if they *were able to commit* to not offering any modification at all. Similarly, results reported in Table 12 show that the lenders could also have raised their expected revenues if they *could commit* to automatically modifying the loan balances whenever the housing prices decline by more than 13 percent. In reality, however, as we mentioned in the discussions following Table 1, only 5.24 percent of the loans were modified. The puzzle is what explains this small presence of loan modifications. Table 13 suggests that the lenders' revenues would have been higher if they did not offer any loan modification; yet Table 12 and many commentators suggest that they should have offered more modification and more automatic modification.

We would like to point out that the results in Tables 12 and 13 are both predicated on the lenders having the ability to commit: In the case of Table 12 the lenders need to be able to commit to automatically reduce the loan balances whenever the housing prices decline by more than 13 percent regardless of whether borrowers have shown any difficulties in making the payments; and, in the case of Table 13, the lenders need to commit to never offering modification to any borrowers regardless of their circumstances. In reality, the lenders do not have the commitment power. We believe that the lack of commitment power is an important contributor to understanding lender behavior and we will investigate this issue in future research.

## 7.4 Quantitative Assessment of the Impact on Borrowers' Ex Ante Expected Welfare

In Table 14, we summarize the borrowers' average expected welfare evaluated at the loan age of one month under the baseline and different counterfactual scenarios. It shows that borrowers are always better off under the automatic modification mortgages of both forms (19) and (20), and with or without cushions. As expected, borrowers are better off under automatic modification mortgages that adjust both payments and balances than under automatic modification mortgages that adjust only payments; also, the borrowers are better off when the automatic adjustment is triggered without cushions (when $\kappa = 1$) than with a cushion (when $\kappa = 1.15$). We also find that borrowers are significantly worse off if the lenders commit to not offering any loan modifications.

| Loan Year of Origination | 2004 | 2005 | 2006 |
|---|---|---|---|
| Baseline | 34.859 | 34.819 | 33.293 |
| Automatic Modification, Just Payment ($\kappa = 1$) | 35.007 | 35.796 | 35.768 |
| Automatic Modification, Both Payment and Balance ($\kappa = 1$) | 35.014 | 35.836 | 37.703 |
| Automatic Modification, Just Payment ($\kappa = 1.15$) | 34.887 | 35.111 | 33.881 |
| Automatic Modification, Both Payment and Balance ($\kappa = 1.15$) | 34.898 | 35.363 | 35.513 |
| No Modification, More Liquidation | 32.576 | 32.076 | 30.304 |
| No Modification, More Waiting | 32.867 | 32.472 | 30.556 |

Table 14: Borrower's Ex Ante Expected Welfare Under the Baseline and Different Counterfactual Scenarios

Notes: (1) Units are in $\$1,000$. (2) The borrowers' expected welfare evaluated at loan age of one month. The expected utility of the borrowers under different scenarios is converted into dollar units via dividing the utility by the estimated coefficient for monthly payments in the utility function.

# 8 Conclusion

One important characteristic of the recent mortgage crisis is the prevalence of subprime mortgages with adjustable interest rates and their high default rates. In this paper, we present a dynamic structural model of subprime adjustable-rate mortgage (ARM) borrowers making payment decisions, taking into account possible consequences of different degrees of delinquency from their lenders. We empirically implement the model using unique data sets that contain information on borrowers' mortgage payment histories, their broad balance sheets, and lender responses.

Our investigation of the factors that drive borrowers' decisions reveals that subprime ARMs are not all alike. For loans originated in 2004 and 2005, which preceded the peak of the housing prices, the interest rate resets associated with ARMs as well as the housing and labor market conditions were not as important in borrowers' delinquency decisions as in their decisions to pay off their loans. For loans originated in 2006, interest rate resets, housing price declines, and worsening labor market conditions all contributed significantly to their high delinquency rates. Counterfactual policy simulations further suggest that even if the LIBOR rates could be lowered to zero by aggressive monetary policies, it would have a limited effect on reducing the delinquency rates. We also examine the effectiveness of automatic modification mortgages under which the monthly payments or the principal balances of the loans are automatically reduced when housing prices decline. We show that such alternative mortgage designs can be effective in reducing both delinquency and foreclosure; and importantly, we find that automatic modification mortgages *with cushions,* which will trigger the monthly payment or principal balance reductions *only when* housing price declines exceed a certain percentage, may result in a Pareto improvement in that borrowers and lenders are both made better off than under the baseline, with much lower delinquency and foreclosure rates.

Our counterfactual analysis also suggests that limited commitment on the part of lenders to loan modification policies may be an important reason for the relatively low rate of modifications observed during the housing crisis. In future research, we plan to model lender behavior explicitly, so that we can have a better understanding of the nature of the lenders' lack of commitment issues, and how policies may be designed to alleviate lenders' lack of commitment power problem. It is also important to consider the general equilibrium effects of alternative mortgages on the housing market, both on the mortgage interest rates the lenders may charge and on the housing market prices, particularly taking into account the spillover effects on property prices due to foreclosed properties.

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# A Appendix

| | At Origination | | | Dynamic Sample | | |
|---|---|---|---|---|---|---|
| Variable | Mean | Median | Std. Dev. | Mean | Median | Std. Dev. |
| Age of the loan (months) | 0 | 0 | 0 | 15 | 12 | 11 |
| Share of 2-year fixed period (%) | 76 | 1 | 43 | 71 | 1 | 45 |
| Prepayment penalty (%) | 0.99 | 1 | 0.09 | 0.88 | 1 | 0.32 |
| Interest-only mortgages (%) | 30 | 0 | 46 | 34 | 0 | 47 |
| Full documentation at origination (%) | 53 | 1 | 50 | 53 | 1 | 50 |
| Purchase loan (%) | 42 | 0 | 49 | 46 | 0 | 50 |
| Credit score | 446 | 445 | 159 | 448 | 457 | 173 |
| Inverse-LTV ratio at origination (%) | 79 | 80 | 11 | 69 | 70 | 14 |
| Annual income ($1,000) | 70 | 66 | 26 | | | |
| Principal balance ($1,000) | 239 | 213 | 127 | 239 | 212 | 131 |
| Current interest rate (%) | 6.78 | 6.70 | 1.09 | 6.95 | 6.75 | 1.40 |
| Remaining mortgage terms (months) | 360 | 360 | 0 | 346 | 349 | 11 |
| Monthly payment ($1,000) | 1.442 | 1.293 | 0.720 | 1.417 | 1.267 | 0.719 |
| Maximum lifetime interest rate (%) | 13.19 | 13.15 | 1.27 | 13.06 | 13.00 | 1.28 |
| Minimum lifetime interest rate (%) | 6.40 | 6.63 | 1.76 | 6.23 | 6.50 | 1.83 |
| Periodic interest rate cap (%) | 1.23 | 1.00 | 0.36 | 1.22 | 1.00 | 0.37 |
| Periodic interest rate floor (%) | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| First rate cap (%) | 2.50 | 3.00 | 0.94 | 2.54 | 3 | 1.02 |
| Margin for adjustable-rate loans (%) | 5.63 | 5.80 | 1.26 | 5.54 | 5.75 | 1.34 |
| 30 days delinquent(%) | 0 | 0 | 0 | 5.46 | 0.0 | 22.72 |
| 60 days delinquent(%) | 0 | 0 | 0 | 1.83 | 0.0 | 13.42 |
| 90 days delinquent(%) | 0 | 0 | 0 | 0.73 | 0.0 | 8.50 |
| 120 days delinquent(%) | 0 | 0 | 0 | 0.57 | 0.0 | 7.51 |
| 150 days delinquent(%) | 0 | 0 | 0 | 0.48 | 0.0 | 6.88 |
| 180 days delinquent(%) | 0 | 0 | 0 | 0.40 | 0.0 | 10.63 |
| More than 180 days delinquent(%) | 0 | 0 | 0 | 1.51 | 0.0 | 12.18 |
| House liquidation (%) | 0 | 0 | 0 | 0.24 | 0.0 | 4.94 |
| Loan modification (%) | 0 | 0 | 0 | 0.54 | 0.0 | 7.30 |
| Deviation local unemployment rates (%) | | | | -1.68 | -1.80 | 1.03 |
| Local housing price growth rates (%) | | | | 0.01 | 0.01 | 1.80 |
| Number of observations | | 6,013 | | | 108,178 | |

Table A1: Summary Statistics for Loans Originated in 2004

| | At Origination | | | Dynamic Sample | | |
|---|---|---|---|---|---|---|
| Variable | Mean | Median | Std. Dev. | Mean | Median | Std. Dev. |
| Age of the loan (months) | 0 | 0 | 0 | 17 | 12 | 11 |
| Share of 2-year fixed period (%) | 83 | 1 | 38 | 80 | 1 | 40 |
| Prepayment penalty (%) | 99 | 1 | 3 | 80 | 1 | 40 |
| Interest-only mortgages (%) | 45 | 0 | 50 | 49 | 0 | 50 |
| Full documentation at origination (%) | 52 | 1 | 50 | 51 | 1 | 50 |
| Purchase loan (%) | 44 | 0 | 50 | 49 | 0 | 50 |
| Credit score | 450 | 450 | 154 | 426 | 450 | 180 |
| Inverse-LTV ratio at origination (%) | 78 | 80 | 11 | 82 | 79 | 20 |
| Annual income ($1,000) | 72 | 67 | 26 | | | |
| Principal balance ($1,000) | 265 | 236 | 142 | 268 | 239 | 141 |
| Current interest rate (%) | 7.04 | 6.90 | 1.02 | 7.31 | 7.00 | 1.35 |
| Remaining mortgage terms (months) | 360 | 360 | 0 | 344 | 346 | 11 |
| Monthly payment ($1,000) | 1.624 | 1.444 | 0.831 | 1.614 | 1.445 | 0.803 |
| Maximum lifetime interest rate (%) | 13.38 | 13.30 | 1.15 | 13.28 | 13.20 | 1.18 |
| Minimum lifetime interest rate (%) | 6.59 | 6.80 | 1.83 | 6.45 | 6.75 | 1.85 |
| Periodic interest rate cap (%) | 1.19 | 1.00 | 0.30 | 1.19 | 1.00 | 0.30 |
| Periodic interest rate floor (%) | 0.03 | 0.00 | 0.18 | 0.03 | 0.00 | 0.19 |
| First rate cap (%) | 2.48 | 3.00 | 0.86 | 2.51 | 3 | 0.90 |
| Margin for adjustable-rate loans (%) | 5.71 | 5.95 | 1.13 | 5.64 | 5.84 | 1.16 |
| 30 days delinquent (%) | 0 | 0 | 0 | 8.96 | 0.0 | 28.57 |
| 60 days delinquent (%) | 0 | 0 | 0 | 4.85 | 0.0 | 21.49 |
| 90 days delinquent (%) | 0 | 0 | 0 | 2.85 | 0.0 | 16.64 |
| 120 days delinquent (%) | 0 | 0 | 0 | 2.29 | 0.0 | 11.30 |
| 150 days delinquent (%) | 0 | 0 | 0 | 1.20 | 0.0 | 10.89 |
| 180 days delinquent (%) | 0 | 0 | 0 | 1.06 | 0.0 | 10.23 |
| More than 180 days delinquent (%) | 0 | 0 | 0 | 4.28 | 0.0 | 20.25 |
| House liquidation (%) | 0 | 0 | 0 | 0.72 | 0.0 | 8.43 |
| Loan modification (%) | 0 | 0 | 0 | 0.28 | 0.0 | 5.24 |
| Deviation local unemployment rates (%) | | | | -1.65 | -1.93 | 1.39 |
| Local housing price growth rates (%) | | | | - 0.005 | - 0.004 | 0.020 |
| Number of observations | | 7,105 | | | 157,544 | |

Table A2: Summary Statistics for Loans Originated in 2005

| | At Origination | | | Dynamic Sample | | |
|---|---|---|---|---|---|---|
| Variable | Mean | Median | Std. Dev. | Mean | Median | Std. Dev. |
| Age of the loan (months) | 0 | 0 | 0 | 15 | 15 | 8.84 |
| Share of 2-yr fixed period (%) | 87 | 1 | 34 | 87 | 1 | 34 |
| Prepayment penalty (%) | 99 | 1 | 2 | 83 | 1 | 77 |
| Interest-only mortgages (%) | 46 | 0 | 50 | 47 | 0 | 50 |
| Full documentation at orig. (%) | 49 | 0 | 50 | 50 | 1 | 50 |
| Purchase loan (%) | 47 | 0 | 50 | 50 | 1 | 50 |
| Credit score | 436 | 435 | 148 | 386 | 383 | 176 |
| Inverse-LTV ratio at origination (%) | 78 | 80 | 12 | 94 | 90 | 24 |
| Annual income ($1,000) | 73 | 67 | 27 | 77 | 75 | 28 |
| Principal balance ($1,000) | 281 | 241 | 159 | 274 | 235 | 153 |
| Current interest rate (%) | 7.99 | 7.88 | 1.06 | 8.02 | 7.94 | 1.14 |
| Remaining mortgage terms (months) | 360 | 360 | 0 | 346 | 346 | 8.84 |
| Monthly payment ($1,000) | 1.922 | 1.661 | 1.061 | 1.869 | 1.634 | 0.995 |
| Maximum lifetime interest rate (%) | 14.30 | 14.24 | 1.20 | 14.23 | 14.12 | 1.18 |
| Minimum lifetime interest rate (%) | 7.49 | 7.75 | 1.87 | 7.42 | 7.65 | 1.84 |
| Periodic interest rate cap (%) | 1.17 | 1.00 | 0.29 | 1.17 | 1.00 | 0.29 |
| Periodic interest rate floor (%) | 0.01 | 0.00 | 0.08 | 0.01 | 0.00 | 0.08 |
| First rate cap (%) | 2.54 | 3.00 | 0.75 | 2.55 | 3 | 0.75 |
| Margin for adjustable-rate loans (%) | 5.92 | 6.00 | 1.07 | 5.90 | 6.00 | 1.07 |
| 30 days delinquent(%) | 0 | 0 | 0 | 8.97 | 0.0 | 28.56 |
| 60 days delinquent(%) | 0 | 0 | 0 | 4.85 | 0.0 | 21.49 |
| 90 days delinquent(%) | 0 | 0 | 0 | 2.85 | 0.0 | 16.64 |
| 120 days delinquent(%) | 0 | 0 | 0 | 2.55 | 0.0 | 15.76 |
| 150 days delinquent(%) | 0 | 0 | 0 | 2.32 | 0.0 | 15.07 |
| 180 days delinquent(%) | 0 | 0 | 0 | 2.11 | 0.0 | 14.37 |
| More than 180 days more delinquent(%) | 0 | 0 | 0 | 6.85 | 0.0 | 25.28 |
| House liquidation (%) | 0 | 0 | 0 | 1.13 | 0.0 | 10.59 |
| Loan modification (%) | 0 | 0 | 0 | 0.49 | 0.0 | 6.97 |
| Deviation local unemployment rates (%) | | | | -1.02 | -1.52 | 1.70 |
| Local housing price growth rates (%) | | | | - 1.58 | - 1.42 | 1.68 |
| Number of observations | | 2,840 | | | 64,308 | |

Table A3: Summary Statistics for Loans Originated in 2006

| Variable | Category 1 Loans ( $d_t = 0, a_t = 0$) | | Category 2 Loans ( $d_t = 1, a_t = 0$) | | Category 3 Loans ( $d_t = 2, a_t = 0$) | |
|---|---|---|---|---|---|---|
| | coeff. | s.d. | coeff. | s.d. | coeff. | s.d. |
| Current Credit Score | 0.0030*** | 0.0017 | 0.0010*** | 0.0002 | -0.0004 | 0.0003 |
| Income at origination ($1,000) | -0.0025 | 0.0017 | -0.0061*** | 0.0018 | -0.0091*** | 0.0019 |
| Loan-to-value | 0.0004 | 0.214 | 0.0004 | 0.0210 | -0.3292 | 0.3997 |
| Loan-to-value at origination | -1.3357*** | 0.3241 | -0.4142* | 0.2230 | 0.3478 | 0.3514 |
| Initial monthly payment ($1,000) | -0.5874*** | 0.0979 | -0.5164*** | 0.1106 | -0.3439*** | 0.1268 |
| Monthly payment ($1,000) | 0.5790*** | 0.0960 | 0.5884*** | 0.195 | 0.5765*** | 0.1254 |
| Dummy for 3-yr fixed period | -0.9434*** | 0.360 | -0.4273*** | 0.1175 | -0.4320*** | 0.1275 |
| Loan age (months) | 1.0312*** | 0.0580 | 0.6360*** | 0.0490 | 0.3970*** | 0.0419 |
| Loan age squared | - 0.0191*** | 0.0012 | -0.01130*** | 0.0010 | -0.0071*** | 0.0009 |
| Dummy for full documentation | 0.1878*** | 0.0640 | 0.1600*** | 0.0672 | 0.1752*** | 0.0718 |
| Deviation local unemp. rate (%) | -0.0443 | 0.0248 | 0.1797*** | 0.0248 | 0.2060*** | 0.0325 |
| Constant | -16.1495*** | 0.7985 | -11.738*** | 0.6887 | -8.8700*** | 0.6275 |
| Number of observations | 78,568 | | 52,154 | | 41,221 | |
| Pseudo-$R^2$ | 0.1955 | | 0.1389 | | 0.0991 | |

Table A4: Lenders' Decisions for Loans in Categories 1-3 (Florida, origination year: 2006)

Notes: (1) Results are from logit regressions where the dependent variable is a dummy for loan modification. (2) ***, ** and * denote statistical significance at 1%, 5%, and 10% respectively. (3) s.d. stands for "standard deviation."

| | Modification | | | Liquidation | |
|---|---|---|---|---|---|
| Variable | coeff. | s.d. | | coeff. | s.d. |
| Current Credit Score | 0.0000 | 0.0003 | | 0.0006*** | 0.0001 |
| Income at origination ($1,000) | -0.1178*** | 0.0009 | | 0.0024*** | 0.0005 |
| Loan-to-value | -1.8278*** | 0.5408 | | 4.6127*** | 0.5897 |
| Loan-to-value at origination | 0.5969*** | 0.1636 | | -1.1419 | 0.1248 |
| Deviation in local unemp. rates (%) | 0.3224 | 0.0528 | | - 0.0987 | 0.0757 |
| Current monthly payment ($1,000) | 0.0141 | 0.0816 | | -0.1620*** | 0.0512 |
| Initial monthly payment ($1,000) | 0.1288 | 0.0830 | | 0.0703 | 0.0514 |
| Loan age (months) | 0.2019*** | 0.0174 | | -0.1291*** | 0.0109 |
| Loan age squared | -0.0038*** | 0.0004 | | 0.0026*** | 0.0002 |
| Months of delinquency | -0.3988*** | 0.1261 | | 0.6474*** | 0.0921 |
| Months of delinquency squared | 0.0132** | 0.0048 | | -0.0170*** | 0.0320 |
| Loan to value ratio × Months of delinquency | 0.4045*** | 0.1061 | | -0.4214*** | 0.0866 |
| Loan to value ratio × Months of delinquency squared | -0.0138** | 0.0046 | | 0.0101*** | 0.0032 |
| Dummy for full documentation | 0.2027*** | 0.0323 | | -0.0715*** | 0.0216 |
| Change in unemp rates × number of late payments | -0.0138 | 0.0107 | | -0.0156 | 0.0107 |
| Change in unemp rates × number of late payments$^2$ | -0.0000 | 0.0005 | | - 0.0008** | 0.0004 |
| Dummy for 4-month delinquency | 0.6252 | 0.4113 | | -3.8769*** | 0.2652 |
| Dummy for 5-month delinquency | 0.73538** | 0.3514 | | -2.8718*** | 0.2029 |
| Dummy for 6-month delinquency | 0.4998 | 0.2973 | | -2.5894*** | 0.1679 |
| Dummy for 7-month delinquency | 0.6295** | 0.2472 | | -1.8006*** | 0.1295 |
| Dummy for 8-month delinquency | 0.4306** | 0.2039 | | -1.1820*** | 0.1003 |
| Dummy for 9-month delinquency | 0.3777** | 0.1661 | | -0.5726*** | 0.0765 |
| Dummy for 10-month delinquency | 0.2491 | 0.1366 | | -0.2683*** | 0.0597 |
| Dummy for 11-month delinquency | 0.1405 | 0.1162 | | -0.0699 | 0.0478 |
| Constant | -5.6849*** | 0.8695 | | -6.0277*** | 0.6546 |
| Number of observations | | | 304,984 | | |
| Pseudo-$R^2$ | | | 0.0933 | | |

Table A5: Lenders' Decisions on Category 4 Loans (Florida, origination year: 2006)

Notes: (1) Results are from multinomial logit regressions where the alternatives are modification, liquidation, and waiting (omitted). (2) ***, ** and * denote statistical significance at 1%, 5%, and 10%, respectively. (3) s.d. stands for "standard deviation."

| Variable | Modification coeff. | s.d. |
|---|---|---|
| Log (initial interest rate) | 0.4131*** | 0.0270 |
| Log (margin rate) | 0.2042*** | 0.0141 |
| Initial balance ($1,000) | 0.0018*** | 0.0006 |
| Remaining balance ($1,000) | -0.0022** | 0.0005 |
| Current credit score | -0.00004** | 0.00002 |
| Income at origination | 0.0004*** | 0.0001 |
| Loan-to-value ratio | 0.2583 *** | 0.0203 |
| Loan-to-value ratio at origination | -0.1266*** | 0.0200 |
| Local unemployment rate deviation | -0.0285*** | 0.0016 |
| Loan age (months) | -0.0103*** | 0.0025 |
| Loan age squared | 0.0003*** | 0.0001 |
| Full documentation | 0.0463*** | 0.0042 |
| Months of delinquency | -0.0209*** | 0.0016 |
| Months of delinquency squared | 0.0001 | 0.0001 |
| Loans originated in Arizona | -0.0176 | 0.0106 |
| Loans originated in California | -0.0136 | 0.0098 |
| Loans originated in Florida | -0.0197** | 0.0098 |
| Constant | 0.7171*** | 0.0719 |
| Number of observations | 18,646 | |
| Pseudo-$R^2$ | 0.2738 | |

Table A6: Lenders' Decisions on Modified Interest rates (origination year: 2006)

Notes: (1) ***, ** and * denote statistical significance at 1%, 5%, and 10%, respectively. (2) s.d. stands for "standard deviation."