Comparing Models of Economic Fluctuations: How Big are the Differences?

Appendix

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1 Results for Alternative Estimation Windows

Table A1: Root MSFEs for Output for 120 Quarter Estimation Window

λ	Horizon	Unrestricted RMSFE	VAR-MINN	RBC	HABIT	ISHOCK	STICKY	STICKY-ACC
0.25	1	0.01	1.14	1.00	***0.99***	1.00	**0.99**	**0.99**
	2	0.02	1.01	0.93	***0.94***	0.93	**0.93**	**0.93**
	4	0.04	*0.88*	0.89	0.93	0.89	0.90	0.91
	8	0.06	*0.82*	0.92	0.96	0.91	0.92	0.93
	12	0.09	*0.81*	0.93	0.96	0.93	0.92	0.93
	16	0.12	*0.80*	0.93	0.95	0.92	0.90	0.92
0.5	1	0.01	1.14	1.00	**0.99**	1.01	**0.99**	**0.99**
	2	0.02	1.01	**0.92**	**0.93**	**0.93**	**0.92**	**0.92**
	4	0.04	0.88	0.88	0.91	0.88	0.88	0.89
	8	0.06	*0.82*	0.90	0.94	0.90	0.90	0.91
	12	0.09	*0.81*	0.92	0.94	0.92	0.89	0.91
	16	0.12	*0.80*	0.91	0.93	0.90	0.88	0.89
1	1	0.01	1.14	1.01	**0.99**	1.02	***0.99***	**0.99**
	2	0.02	1.01	**0.92**	**0.92**	**0.93**	**0.91**	***0.91***
	4	0.04	0.88	**0.87**	0.89	0.88	***0.86***	**0.87**
	8	0.06	*0.82*	0.89	0.93	0.89	0.87	0.89
	12	0.09	*0.81*	0.90	0.93	0.90	0.87	0.89
	16	0.12	*0.80*	0.89	0.91	0.89	0.85	0.87
5	1	0.01	1.14	1.04	1.00	1.04	***0.99***	**1.00**
	2	0.02	1.01	**0.93**	**0.89**	**0.94**	**0.88**	***0.88***
	4	0.04	0.88	0.86	0.85	0.88	**0.82**	***0.82***
	8	0.06	*0.82*	0.86	0.89	0.87	0.82	0.83
	12	0.09	*0.81*	0.87	0.89	0.88	0.82	0.83
	16	0.12	*0.80*	0.86	0.88	0.87	0.81	0.82
10	1	*0.01*	1.14	1.08	1.00	1.06	1.01	1.02
	2	0.02	1.01	**0.94**	**0.88**	**0.95**	**0.88**	***0.87***
	4	0.04	0.88	**0.86**	**0.83**	**0.87**	**0.81**	***0.80***
	8	0.06	0.82	0.85	0.87	0.86	**0.80**	***0.80***
	12	0.09	0.81	0.86	0.87	0.87	***0.80***	**0.80**
	16	0.12	0.80	0.85	0.87	0.86	***0.79***	**0.79**
20	1	*0.01*	1.14	1.13	1.02	1.08	1.05	1.07
	2	0.02	1.01	**0.95**	**0.86**	**0.95**	**0.87**	***0.86***
	4	0.04	0.88	**0.85**	**0.81**	**0.87**	**0.79**	***0.77***
	8	0.06	0.82	0.83	0.84	0.85	**0.78**	***0.77***
	12	0.09	0.81	0.84	0.85	0.86	**0.78**	***0.78***
	16	0.12	0.80	0.83	0.85	0.85	***0.77***	***0.77***
∞	1	*0.01*	1.14	4.88	4.32	4.88	90.24	2.83
	2	*0.02*	1.01	2.77	2.34	2.77	30.67	1.61
	4	0.04	0.88	1.27	1.02	1.28	41.79	***0.86***
	8	0.06	0.82	**0.65**	***0.63***	**0.65**	28.78	**0.65**
	12	0.09	0.81	**0.57**	**0.64**	***0.57***	25.15	**0.67**
	16	0.12	0.80	**0.59**	**0.67**	***0.59***	24.27	**0.69**

See notes to table 2 in body of paper.

Table A2: Root MSFEs for Investment for 120 Quarter Estimation Window

λ	Horizon	Unrestricted RMSFE	VAR-MINN	RBC	HABIT	ISHOCK	STICKY	STICKY-ACC
0.25	1	0.05	1.15	**0.99**	***0.97***	**0.99**	**0.98**	**0.98**
	2	0.08	1.01	**0.91**	**0.91**	***0.90***	**0.92**	**0.91**
	4	0.12	0.88	**0.84**	0.89	***0.83***	**0.87**	**0.87**
	8	0.16	*0.83*	0.89	0.97	0.85	0.93	0.94
	12	0.19	*0.83*	0.92	0.98	0.88	0.93	0.94
	16	0.23	*0.79*	0.88	0.95	0.84	0.89	0.90
0.5	1	0.05	1.15	1.00	***0.97***	1.00	**0.98**	**0.98**
	2	0.08	1.01	**0.91**	***0.90***	**0.91**	**0.91**	**0.91**
	4	0.12	0.88	**0.84**	**0.87**	***0.83***	**0.86**	**0.86**
	8	0.16	*0.83*	0.87	0.96	0.85	0.92	0.93
	12	0.19	*0.83*	0.90	0.97	0.87	0.92	0.93
	16	0.23	*0.79*	0.86	0.93	0.82	0.87	0.88
1	1	0.05	1.15	1.01	***0.97***	1.01	**0.99**	**0.99**
	2	0.08	1.01	**0.92**	***0.90***	0.92	**0.91**	**0.91**
	4	0.12	0.88	**0.84**	**0.86**	***0.84***	**0.85**	**0.86**
	8	0.16	*0.83*	0.87	0.95	0.85	0.91	0.92
	12	0.19	*0.83*	0.89	0.96	0.87	0.91	0.92
	16	0.23	*0.79*	0.84	0.92	0.81	0.85	0.87
5	1	0.05	1.15	1.05	***0.99***	1.04	1.00	1.01
	2	0.08	1.01	**0.96**	***0.90***	**0.95**	**0.92**	**0.92**
	4	0.12	0.88	**0.87**	***0.84***	**0.87**	**0.85**	**0.86**
	8	0.16	*0.83*	0.87	0.92	0.86	0.89	0.90
	12	0.19	*0.83*	0.88	0.94	0.86	0.88	0.90
	16	0.23	*0.79*	0.82	0.89	0.79	0.83	0.84
10	1	*0.05*	1.15	1.07	1.00	1.05	1.01	1.02
	2	0.08	1.01	**0.98**	***0.91***	**0.97**	**0.92**	**0.93**
	4	0.12	0.88	0.89	***0.84***	0.88	**0.85**	**0.86**
	8	0.16	*0.83*	0.87	0.91	0.86	0.88	0.89
	12	0.19	*0.83*	0.87	0.93	0.86	0.87	0.88
	16	0.23	0.79	0.81	0.89	**0.78**	0.82	0.83
20	1	*0.05*	1.15	1.09	1.02	1.07	1.01	1.03
	2	0.08	1.01	1.00	***0.92***	**0.98**	**0.93**	**0.94**
	4	0.12	0.88	0.89	***0.85***	**0.88**	**0.85**	**0.86**
	8	0.16	*0.83*	0.86	0.89	0.86	0.87	0.87
	12	0.19	*0.83*	0.86	0.92	0.85	0.87	0.87
	16	0.23	0.79	0.80	0.88	**0.77**	0.81	0.82
∞	1	*0.05*	1.15	1.24	1.19	1.22	10.52	1.13
	2	*0.08*	1.01	1.10	1.05	1.08	11.65	1.02
	4	0.12	*0.88*	0.93	0.90	0.90	6.84	0.90
	8	0.16	0.83	**0.79**	**0.81**	***0.76***	3.84	0.88
	12	0.19	0.83	**0.70**	**0.78**	***0.68***	2.86	0.87
	16	0.23	0.79	**0.66**	**0.72**	***0.65***	1.39	0.83

See notes to table 2 in body of paper.

Table A3: Root MSFEs for Hours for 120 Quarter Estimation Window

λ	Horizon	Unrestricted RMSFE	VAR-MINN	RBC	HABIT	ISHOCK	STICKY	STICKY-ACC
0.25	1	0.01	1.17	**0.98**	**0.98**	***0.97***	1.04	1.02
	2	0.02	1.11	**0.93**	**0.94**	***0.91***	1.01	**0.98**
	4	0.03	1.03	**0.87**	**0.91**	***0.85***	**0.96**	**0.93**
	8	0.05	0.99	**0.89**	**0.94**	***0.86***	**0.95**	**0.94**
	12	0.06	1.00	**0.89**	**0.94**	***0.85***	**0.95**	**0.93**
	16	0.07	0.97	**0.85**	**0.91**	***0.80***	**0.93**	**0.91**
0.5	1	0.01	1.17	**0.98**	**0.98**	***0.97***	1.06	1.03
	2	0.02	1.11	**0.92**	**0.93**	***0.90***	1.02	**0.98**
	4	0.03	1.03	**0.86**	**0.90**	***0.83***	**0.96**	**0.93**
	8	0.05	0.99	**0.87**	**0.92**	***0.83***	**0.95**	**0.92**
	12	0.06	1.00	**0.86**	**0.92**	***0.82***	**0.94**	**0.92**
	16	0.07	0.97	**0.81**	**0.89**	***0.76***	**0.92**	**0.89**
1	1	0.01	1.17	**0.99**	**0.99**	***0.97***	1.07	1.04
	2	0.02	1.11	**0.92**	**0.93**	***0.89***	1.03	**0.99**
	4	0.03	1.03	**0.85**	**0.89**	***0.82***	**0.97**	**0.93**
	8	0.05	0.99	**0.85**	**0.91**	***0.82***	**0.95**	**0.92**
	12	0.06	1.00	**0.84**	**0.90**	***0.80***	**0.94**	**0.92**
	16	0.07	0.97	**0.79**	**0.87**	***0.73***	**0.92**	**0.89**
5	1	0.01	1.17	1.03	1.00	***0.99***	1.11	1.08
	2	0.02	1.11	**0.96**	**0.93**	***0.91***	1.08	1.02
	4	0.03	1.03	**0.88**	**0.89**	***0.83***	1.02	**0.97**
	8	0.05	0.99	**0.85**	**0.90**	***0.81***	0.99	**0.95**
	12	0.06	1.00	**0.83**	**0.90**	***0.77***	**0.99**	**0.95**
	16	0.07	0.97	**0.76**	**0.86**	***0.70***	**0.96**	**0.92**
10	1	*0.01*	1.17	1.06	1.01	1.01	1.13	1.10
	2	0.02	1.11	**0.99**	**0.94**	***0.93***	1.11	1.05
	4	0.03	1.03	**0.91**	**0.89**	***0.85***	1.05	**1.00**
	8	0.05	0.99	**0.86**	**0.90**	***0.81***	1.02	**0.98**
	12	0.06	1.00	**0.83**	**0.90**	***0.77***	1.02	**0.98**
	16	0.07	0.97	**0.77**	**0.87**	***0.70***	0.99	**0.95**
20	1	*0.01*	1.17	1.09	1.02	1.03	1.16	1.13
	2	0.02	1.11	1.03	***0.96***	**0.96**	1.15	1.10
	4	0.03	1.03	**0.94**	**0.91**	***0.87***	1.09	1.05
	8	0.05	0.99	**0.87**	**0.91**	***0.82***	1.07	1.02
	12	0.06	1.00	**0.84**	**0.92**	***0.78***	1.06	1.02
	16	0.07	0.97	**0.77**	**0.88**	***0.70***	1.02	0.99
∞	1	*0.01*	1.17	1.43	1.56	1.45	31.62	1.55
	2	*0.02*	1.11	1.45	1.59	1.49	31.53	1.61
	4	*0.03*	1.03	1.35	1.43	1.40	8.70	1.53
	8	0.05	*0.99*	1.23	1.25	1.27	3.70	1.40
	12	*0.06*	1.00	1.19	1.19	1.23	6.41	1.33
	16	0.07	*0.97*	1.15	1.13	1.19	6.57	1.25

See notes to table 2 in body of paper.

Table A4: Root MSFEs for Consumption for 120 Quarter Estimation Window

λ	Horizon	Unrestricted RMSFE	VAR-MINN	RBC	HABIT	ISHOCK	STICKY	STICKY-ACC
0.25	1	*0.01*	1.04	1.01	1.00	1.02	1.01	1.01
	2	0.01	*0.95*	0.99	0.99	0.99	0.97	0.97
	4	0.03	*0.87*	0.97	0.99	0.97	0.94	0.95
	8	0.06	*0.83*	0.97	0.99	0.97	0.93	0.94
	12	0.09	*0.83*	0.98	0.98	0.98	0.92	0.94
	16	0.12	*0.83*	0.97	0.98	0.97	0.92	0.93
0.5	1	*0.01*	1.04	1.01	1.01	1.02	1.01	1.01
	2	0.01	*0.95*	0.99	0.99	1.01	0.96	0.97
	4	0.03	*0.87*	0.97	0.99	0.98	0.92	0.94
	8	0.06	*0.83*	0.97	0.98	0.98	0.91	0.93
	12	0.09	*0.83*	0.97	0.97	0.98	0.90	0.92
	16	0.12	*0.83*	0.97	0.97	0.98	0.90	0.92
1	1	*0.01*	1.04	1.02	1.01	1.03	1.01	1.02
	2	0.01	0.95	1.00	0.99	1.02	0.95	0.96
	4	0.03	*0.87*	0.97	0.98	0.99	0.91	0.93
	8	0.06	*0.83*	0.97	0.97	0.99	0.89	0.91
	12	0.09	*0.83*	0.97	0.96	0.99	0.88	0.90
	16	0.12	*0.83*	0.96	0.96	0.99	0.88	0.90
5	1	*0.01*	1.04	1.03	1.01	1.05	1.01	1.02
	2	0.01	0.95	1.02	0.98	1.05	***0.94***	0.95
	4	0.03	*0.87*	0.99	0.97	1.03	0.88	0.90
	8	0.06	*0.83*	0.97	0.96	1.01	0.86	0.88
	12	0.09	*0.83*	0.96	0.95	1.00	0.85	0.87
	16	0.12	*0.83*	0.95	0.94	1.00	0.84	0.86
10	1	0.01	1.04	1.04	1.00	1.05	1.01	1.02
	2	0.01	0.95	1.03	0.98	1.06	***0.93***	**0.95**
	4	0.03	*0.87*	0.99	0.97	1.03	0.87	0.89
	8	0.06	*0.83*	0.96	0.96	1.01	0.84	0.86
	12	0.09	*0.83*	0.96	0.95	1.00	0.83	0.85
	16	0.12	0.83	0.95	0.94	1.00	***0.83***	0.84
20	1	0.01	1.04	1.05	1.00	1.06	1.01	1.02
	2	0.01	0.95	1.03	0.97	1.07	**0.93**	**0.94**
	4	0.03	0.87	0.99	0.96	1.04	**0.86**	0.88
	8	0.06	*0.83*	0.96	0.95	1.01	0.83	0.84
	12	0.09	0.83	0.95	0.94	1.00	**0.82**	0.83
	16	0.12	0.83	0.94	0.94	0.99	**0.81**	**0.82**
∞	1	*0.01*	1.04	1.16	1.06	1.20	1.08	1.05
	2	0.01	0.95	0.95	**0.92**	0.99	***0.88***	**0.89**
	4	0.03	0.87	***0.75***	**0.80**	**0.77**	**0.87**	**0.76**
	8	0.06	0.83	***0.67***	**0.76**	**0.68**	**0.78**	**0.71**
	12	0.09	0.83	**0.65**	**0.75**	***0.65***	**0.76**	**0.71**
	16	0.12	0.83	**0.66**	**0.75**	***0.65***	**0.79**	**0.72**

See notes to table 2 in body of paper.

Table A5: Root MSFEs for Output for 140 Quarter Estimation Window

λ	Horizon	Unrestricted RMSFE	VAR-MINN	RBC	HABIT	ISHOCK	STICKY	STICKY-ACC
0.25	1	0.01	1.11	**0.99**	***0.98***	**1.00**	**1.00**	**1.00**
	2	0.02	1.00	**0.96**	**0.96**	***0.96***	**0.97**	**0.97**
	4	0.04	*0.89*	0.94	0.97	0.93	0.96	0.96
	8	0.07	*0.83*	0.94	0.98	0.94	0.95	0.97
	12	0.09	*0.84*	0.96	0.98	0.95	0.95	0.97
	16	0.11	*0.86*	0.97	0.98	0.96	0.95	0.97
0.5	1	0.01	1.11	**0.99**	***0.97***	**1.00**	**0.99**	**0.99**
	2	0.02	1.00	***0.95***	**0.95**	**0.95**	**0.96**	**0.96**
	4	0.04	*0.89*	0.92	0.96	0.92	0.94	0.94
	8	0.07	*0.83*	0.93	0.97	0.92	0.93	0.94
	12	0.09	*0.84*	0.95	0.97	0.94	0.94	0.95
	16	0.11	*0.86*	0.96	0.98	0.95	0.94	0.95
1	1	0.01	1.11	**0.99**	***0.97***	**1.00**	**0.99**	**0.99**
	2	0.02	1.00	***0.94***	**0.94**	**0.95**	**0.94**	**0.94**
	4	0.04	*0.89*	0.91	0.94	0.91	0.91	0.92
	8	0.07	*0.83*	0.91	0.96	0.91	0.91	0.92
	12	0.09	*0.84*	0.93	0.96	0.93	0.92	0.93
	16	0.11	*0.86*	0.94	0.97	0.94	0.92	0.93
5	1	0.01	1.11	**0.98**	**0.95**	1.01	**0.95**	***0.95***
	2	0.02	1.00	**0.92**	**0.91**	**0.94**	**0.89**	***0.88***
	4	0.04	0.89	**0.88**	0.91	0.90	**0.86**	***0.85***
	8	0.07	*0.83*	0.87	0.92	0.88	0.86	0.86
	12	0.09	*0.84*	0.89	0.94	0.91	0.87	0.88
	16	0.11	*0.86*	0.91	0.94	0.92	0.88	0.88
10	1	0.01	1.11	**0.98**	***0.93***	1.00	**0.94**	**0.93**
	2	0.02	1.00	**0.91**	**0.88**	**0.94**	**0.86**	***0.84***
	4	0.04	0.89	**0.87**	**0.89**	**0.89**	**0.83**	***0.82***
	8	0.07	0.83	0.85	0.90	**0.87**	**0.83**	***0.82***
	12	0.09	*0.84*	0.88	0.92	0.89	0.85	0.85
	16	0.11	0.86	0.90	0.93	0.91	**0.86**	***0.86***
20	1	0.01	1.11	**1.00**	***0.91***	1.00	**0.94**	**0.94**
	2	0.02	1.00	**0.90**	**0.85**	**0.93**	**0.83**	***0.80***
	4	0.04	0.89	**0.85**	**0.86**	**0.88**	**0.80**	***0.78***
	8	0.07	0.83	0.83	0.88	0.85	**0.80**	***0.79***
	12	0.09	0.84	0.86	0.90	0.88	**0.83**	***0.82***
	16	0.11	0.86	0.88	0.92	0.90	**0.84**	***0.83***
∞	1	*0.01*	1.11	5.21	4.87	5.21	98.52	3.04
	2	*0.02*	1.00	2.50	2.19	2.50	30.96	1.39
	4	0.04	0.89	0.96	**0.77**	0.96	42.65	***0.65***
	8	0.07	0.83	**0.53**	**0.56**	***0.53***	33.34	**0.60**
	12	0.09	0.84	**0.57**	**0.66**	***0.57***	36.12	**0.69**
	16	0.11	0.86	**0.64**	**0.72**	***0.64***	41.99	**0.75**

See notes to table 2 in body of paper.

Table A6: Root MSFEs for Investment for 140 Quarter Estimation Window

λ	Horizon	Unrestricted RMSFE	VAR-MINN	RBC	HABIT	ISHOCK	STICKY	STICKY-ACC
0.25	1	0.04	1.23	1.01	***0.99***	1.01	1.02	1.02
	2	0.06	1.13	**0.97**	**0.98**	***0.96***	1.00	1.00
	4	0.10	0.97	**0.91**	**0.97**	***0.89***	0.97	0.98
	8	0.16	*0.82*	0.89	0.98	0.84	0.95	0.97
	12	0.20	*0.81*	0.91	0.98	0.86	0.94	0.95
	16	0.23	*0.83*	0.92	0.97	0.88	0.93	0.94
0.5	1	0.04	1.23	1.02	***0.99***	1.02	1.03	1.02
	2	0.06	1.13	**0.97**	**0.97**	***0.96***	1.00	1.00
	4	0.10	0.97	**0.90**	**0.96**	***0.88***	**0.97**	**0.97**
	8	0.16	*0.82*	0.86	0.97	0.82	0.94	0.94
	12	0.20	*0.81*	0.88	0.97	0.84	0.92	0.93
	16	0.23	*0.83*	0.89	0.96	0.85	0.91	0.92
1	1	0.04	1.23	1.04	***0.99***	1.03	1.04	1.04
	2	0.06	1.13	**0.98**	***0.97***	**0.97**	1.00	1.00
	4	0.10	0.97	**0.90**	**0.95**	***0.88***	**0.95**	**0.96**
	8	0.16	0.82	0.85	0.95	***0.81***	0.92	0.93
	12	0.20	*0.81*	0.86	0.95	0.82	0.90	0.91
	16	0.23	*0.83*	0.87	0.95	0.83	0.89	0.90
5	1	*0.04*	1.23	1.10	1.01	1.07	1.05	1.06
	2	0.06	1.13	1.03	***0.97***	1.00	1.00	1.01
	4	0.10	0.97	**0.92**	**0.93**	***0.90***	**0.93**	**0.94**
	8	0.16	0.82	0.82	0.91	***0.80***	0.88	0.88
	12	0.20	0.81	0.82	0.91	***0.79***	0.87	0.87
	16	0.23	0.83	0.83	0.92	***0.80***	0.86	0.87
10	1	0.04	1.23	1.13	1.02	1.09	1.06	1.08
	2	0.06	1.13	1.06	**0.97**	1.02	1.01	1.02
	4	0.10	0.97	**0.94**	**0.92**	**0.91**	**0.93**	**0.94**
	8	0.16	0.82	**0.81**	0.88	**0.79**	0.86	0.86
	12	0.20	0.81	**0.80**	0.90	**0.78**	0.86	0.86
	16	0.23	0.83	**0.82**	0.91	**0.79**	0.85	0.85
20	1	*0.04*	1.23	1.15	1.04	1.12	1.06	1.10
	2	0.06	1.13	1.08	***0.99***	1.04	1.02	1.04
	4	0.10	0.97	**0.94**	**0.92**	***0.92***	**0.93**	**0.94**
	8	0.16	0.82	**0.80**	0.86	***0.79***	0.85	0.85
	12	0.20	0.81	**0.79**	0.88	***0.77***	0.84	0.84
	16	0.23	0.83	**0.80**	0.89	***0.78***	0.84	0.84
∞	1	*0.04*	1.23	1.32	1.28	1.30	11.74	1.24
	2	*0.06*	1.13	1.20	1.17	1.17	13.14	1.18
	4	0.10	0.97	0.98	0.98	***0.95***	6.89	1.04
	8	0.16	0.82	**0.74**	**0.78**	***0.71***	2.44	0.87
	12	0.20	0.81	**0.68**	**0.74**	***0.66***	3.00	0.85
	16	0.23	0.83	***0.69***	**0.74**	**0.69**	2.28	0.85

See notes to table 2 in body of paper.

Table A7: Root MSFEs for Hours for 140 Quarter Estimation Window

λ	Horizon	Unrestricted RMSFE	VAR-MINN	RBC	HABIT	ISHOCK	STICKY	STICKY-ACC
0.25	1	0.01	1.27	1.01	1.01	***0.99***	1.08	1.06
	2	0.01	1.26	**0.98**	**0.99**	***0.96***	1.08	1.05
	4	0.03	1.17	**0.95**	**0.98**	***0.92***	1.04	1.01
	8	0.04	1.08	**0.94**	**0.98**	***0.91***	1.02	1.00
	12	0.05	1.07	**0.94**	**0.98**	***0.91***	1.01	1.00
	16	0.06	1.06	**0.93**	**0.97**	***0.91***	1.00	0.99
0.5	1	0.01	1.27	1.01	1.01	***0.98***	1.11	1.07
	2	0.01	1.26	**0.98**	**0.99**	***0.94***	1.10	1.06
	4	0.03	1.17	**0.93**	**0.97**	***0.90***	1.05	1.02
	8	0.04	1.08	**0.92**	**0.97**	***0.89***	1.03	**1.00**
	12	0.05	1.07	**0.92**	**0.97**	***0.89***	1.02	**0.99**
	16	0.06	1.06	**0.91**	**0.96**	***0.87***	1.01	**0.98**
1	1	0.01	1.27	1.02	1.02	***0.97***	1.13	1.09
	2	0.01	1.26	**0.98**	**1.00**	***0.93***	1.12	1.07
	4	0.03	1.17	**0.93**	**0.97**	***0.88***	1.07	1.03
	8	0.04	1.08	**0.91**	**0.97**	***0.87***	1.04	1.01
	12	0.05	1.07	**0.90**	**0.96**	***0.86***	1.03	**1.00**
	16	0.06	1.06	**0.88**	**0.95**	***0.84***	1.01	**0.98**
5	1	0.01	1.27	1.07	1.04	***1.00***	1.18	1.14
	2	0.01	1.26	1.03	1.02	***0.94***	1.19	1.12
	4	0.03	1.17	**0.96**	**0.99**	***0.88***	1.13	1.07
	8	0.04	1.08	**0.91**	**0.97**	***0.85***	1.08	1.03
	12	0.05	1.07	**0.88**	**0.96**	***0.82***	1.06	1.02
	16	0.06	1.06	**0.85**	**0.93**	***0.78***	1.04	**0.99**
10	1	*0.01*	1.27	1.11	1.06	1.03	1.20	1.17
	2	0.01	1.26	1.07	1.03	***0.97***	1.22	1.16
	4	0.03	1.17	**0.99**	**1.00**	***0.90***	1.17	1.11
	8	0.04	1.08	**0.92**	**0.98**	***0.85***	1.11	1.06
	12	0.05	1.07	**0.88**	**0.96**	***0.82***	1.09	1.04
	16	0.06	1.06	**0.84**	**0.94**	***0.77***	1.06	1.02
20	1	*0.01*	1.27	1.15	1.08	1.06	1.23	1.20
	2	*0.01*	1.26	1.12	1.06	1.01	1.26	1.21
	4	0.03	1.17	1.02	1.02	***0.94***	1.21	1.16
	8	0.04	1.08	**0.93**	**0.99**	***0.86***	1.15	1.10
	12	0.05	1.07	**0.88**	**0.97**	***0.82***	1.12	1.07
	16	0.06	1.06	**0.84**	**0.95**	***0.77***	1.09	1.05
∞	1	*0.01*	1.27	1.54	1.70	1.56	34.39	1.70
	2	*0.01*	1.26	1.62	1.79	1.65	35.98	1.84
	4	*0.03*	1.17	1.48	1.58	1.52	10.31	1.72
	8	*0.04*	1.08	1.29	1.32	1.32	5.97	1.50
	12	*0.05*	1.07	1.22	1.23	1.26	11.22	1.39
	16	*0.06*	1.06	1.18	1.18	1.22	16.48	1.30

See notes to table 2 in body of paper.

Table A8: Root MSFEs for Consumption for 140 Quarter Estimation Window

λ	Horizon	Unrestricted RMSFE	VAR-MINN	RBC	HABIT	ISHOCK	STICKY	STICKY-ACC
0.25	1	0.01	**0.98**	0.99	0.99	0.99	0.98	0.99
	2	0.01	**0.93**	0.97	0.99	0.98	0.96	0.97
	4	0.03	**0.87**	0.97	0.99	0.97	0.95	0.96
	8	0.06	**0.86**	0.98	1.00	0.97	0.95	0.97
	12	0.09	**0.87**	0.99	1.00	0.99	0.95	0.97
	16	0.11	**0.88**	0.99	1.00	0.99	0.95	0.97
0.5	1	0.01	0.98	0.99	0.99	1.00	***0.98***	0.98
	2	0.01	*0.93*	0.97	0.99	0.98	0.95	0.96
	4	0.03	*0.87*	0.96	0.99	0.97	0.93	0.95
	8	0.06	*0.86*	0.97	0.99	0.98	0.94	0.95
	12	0.09	*0.87*	0.98	0.99	0.99	0.94	0.95
	16	0.11	*0.88*	0.99	0.99	1.00	0.94	0.95
1	1	0.01	0.98	0.99	0.99	1.01	**0.97**	**0.98**
	2	0.01	*0.93*	0.97	0.98	1.00	0.93	0.94
	4	0.03	*0.87*	0.96	0.98	0.98	0.91	0.93
	8	0.06	*0.86*	0.96	0.99	0.98	0.92	0.93
	12	0.09	*0.87*	0.98	0.99	1.00	0.92	0.93
	16	0.11	*0.88*	0.98	0.99	1.00	0.92	0.93
5	1	0.01	0.98	1.01	0.98	1.03	**0.96**	**0.97**
	2	0.01	0.93	0.99	0.97	1.04	**0.90**	**0.91**
	4	0.03	*0.87*	0.96	0.97	1.00	0.87	0.89
	8	0.06	*0.86*	0.95	0.97	0.99	0.87	0.89
	12	0.09	*0.87*	0.97	0.98	1.00	0.88	0.89
	16	0.11	*0.88*	0.97	0.98	1.01	0.88	0.89
10	1	0.01	0.98	1.01	0.98	1.04	***0.96***	**0.97**
	2	0.01	0.93	1.00	0.96	1.05	***0.89***	**0.90**
	4	0.03	0.87	0.96	0.96	1.00	***0.86***	0.87
	8	0.06	*0.86*	0.95	0.97	0.99	0.86	0.87
	12	0.09	0.87	0.96	0.97	1.00	***0.86***	0.87
	16	0.11	0.88	0.97	0.97	1.01	***0.87***	**0.88**
20	1	0.01	0.98	1.02	0.97	1.05	***0.95***	**0.97**
	2	0.01	0.93	1.01	0.96	1.06	**0.88**	**0.89**
	4	0.03	0.87	0.96	0.95	1.01	**0.84**	**0.86**
	8	0.06	0.86	0.94	0.96	0.99	**0.84**	**0.85**
	12	0.09	0.87	0.95	0.97	1.00	**0.85**	**0.85**
	16	0.11	0.88	0.96	0.97	1.00	**0.85**	**0.86**
∞	1	0.01	0.98	1.02	***0.96***	1.05	0.99	**0.96**
	2	0.01	0.93	**0.80**	**0.79**	**0.82**	***0.76***	**0.80**
	4	0.03	0.87	***0.66***	**0.73**	**0.66**	**0.85**	**0.72**
	8	0.06	0.86	**0.65**	**0.74**	***0.64***	**0.78**	**0.70**
	12	0.09	0.87	**0.67**	**0.76**	***0.65***	**0.82**	**0.73**
	16	0.11	0.88	**0.69**	**0.78**	***0.68***	**0.84**	**0.75**

See notes to table 2 in body of paper.

Table A9: Root MSFEs for Output for 180 Quarter Estimation Window

λ	Horizon	Unrestricted RMSFE	VAR-MINN	RBC	HABIT	ISHOCK	STICKY	STICKY-ACC
0.25	1	0.01	0.97	**0.94**	**0.95**	***0.92***	**0.94**	**0.94**
	2	0.02	0.89	0.90	0.94	***0.89***	0.96	0.96
	4	0.03	*0.85*	0.91	0.96	0.90	0.99	0.99
	8	0.07	*0.85*	0.94	0.97	0.93	0.99	1.00
	12	0.10	*0.87*	0.96	0.98	0.95	0.99	0.99
	16	0.12	*0.89*	0.97	0.98	0.96	0.98	0.98
0.5	1	0.01	0.97	**0.92**	**0.93**	***0.91***	**0.91**	**0.92**
	2	0.02	0.89	**0.87**	**0.92**	***0.87***	0.94	0.94
	4	0.03	*0.85*	0.88	0.94	0.88	0.97	0.97
	8	0.07	*0.85*	0.91	0.96	0.91	0.98	0.98
	12	0.10	*0.87*	0.94	0.97	0.93	0.97	0.98
	16	0.12	*0.89*	0.95	0.97	0.95	0.96	0.97
1	1	0.01	0.97	**0.90**	**0.90**	**0.90**	***0.88***	**0.89**
	2	0.02	0.89	***0.84***	**0.89**	**0.85**	0.90	0.90
	4	0.03	0.85	***0.85***	0.92	0.85	0.94	0.94
	8	0.07	*0.85*	0.89	0.94	0.89	0.95	0.96
	12	0.10	*0.87*	0.92	0.95	0.92	0.95	0.96
	16	0.12	0.89	0.93	0.96	0.93	0.95	0.95
5	1	0.01	0.97	**0.86**	**0.85**	**0.87**	***0.78***	**0.80**
	2	0.02	0.89	***0.74***	**0.81**	**0.78**	**0.77**	**0.78**
	4	0.03	0.85	***0.76***	0.85	**0.79**	**0.83**	**0.83**
	8	0.07	0.85	***0.81***	0.89	**0.83**	0.87	0.87
	12	0.10	0.87	***0.86***	0.91	0.87	0.89	0.89
	16	0.12	0.89	***0.88***	0.93	0.90	0.90	0.90
10	1	0.01	0.97	**0.86**	**0.83**	**0.86**	***0.76***	**0.79**
	2	0.02	0.89	***0.68***	**0.78**	**0.74**	**0.70**	**0.71**
	4	0.03	0.85	***0.71***	**0.82**	**0.76**	**0.77**	**0.77**
	8	0.07	0.85	***0.77***	0.86	**0.81**	**0.83**	**0.83**
	12	0.10	0.87	***0.83***	0.89	**0.85**	**0.86**	**0.86**
	16	0.12	0.89	***0.86***	0.91	**0.88**	**0.88**	**0.88**
20	1	0.01	0.97	**0.92**	**0.82**	**0.86**	***0.80***	**0.82**
	2	0.02	0.89	**0.63**	**0.74**	**0.70**	***0.63***	**0.63**
	4	0.03	0.85	***0.65***	**0.78**	**0.71**	**0.70**	**0.70**
	8	0.07	0.85	***0.73***	**0.83**	**0.77**	**0.78**	**0.77**
	12	0.10	0.87	***0.80***	**0.87**	**0.83**	**0.83**	**0.82**
	16	0.12	0.89	***0.84***	**0.90**	**0.86**	**0.85**	**0.85**
∞	1	0.01	*0.97*	4.31	3.92	4.31	77.52	2.62
	2	0.02	*0.89*	1.80	1.54	1.80	23.35	1.06
	4	0.03	0.85	**0.55**	**0.49**	**0.56**	26.06	***0.46***
	8	0.07	0.85	***0.44***	**0.54**	**0.44**	16.67	**0.56**
	12	0.10	0.87	**0.58**	**0.67**	***0.58***	14.80	**0.69**
	16	0.12	0.89	**0.67**	**0.74**	***0.67***	14.84	**0.77**

See notes to table 2 in body of paper.

Table A10: Root MSFEs for Investment for 180 Quarter Estimation Window

λ	Horizon	Unrestricted RMSFE	VAR-MINN	RBC	HABIT	ISHOCK	STICKY	STICKY-ACC
0.25	1	0.04	1.00	**0.91**	**0.95**	***0.88***	**0.94**	**0.95**
	2	0.07	0.91	**0.89**	0.96	***0.83***	0.97	0.97
	4	0.13	0.83	0.88	0.97	***0.82***	0.98	0.98
	8	0.22	*0.79*	0.88	0.96	0.83	0.96	0.97
	12	0.27	*0.82*	0.91	0.97	0.87	0.95	0.96
	16	0.30	*0.85*	0.93	0.97	0.89	0.95	0.95
0.5	1	0.04	1.00	**0.90**	**0.93**	***0.86***	**0.94**	**0.93**
	2	0.07	0.91	**0.86**	0.94	***0.81***	0.95	0.95
	4	0.13	0.83	0.85	0.95	***0.79***	0.96	0.96
	8	0.22	*0.79*	0.85	0.95	0.80	0.94	0.95
	12	0.27	*0.82*	0.88	0.95	0.83	0.93	0.94
	16	0.30	*0.85*	0.90	0.96	0.85	0.93	0.93
1	1	0.04	1.00	**0.89**	**0.91**	**0.85**	**0.93**	**0.93**
	2	0.07	0.91	**0.85**	0.92	**0.80**	0.93	0.93
	4	0.13	0.83	**0.82**	0.93	**0.77**	0.94	0.94
	8	0.22	0.79	0.82	0.93	**0.77**	0.92	0.92
	12	0.27	0.82	0.85	0.93	**0.81**	0.91	0.92
	16	0.30	0.85	0.87	0.94	**0.83**	0.91	0.91
5	1	0.04	1.00	**0.90**	**0.86**	**0.86**	**0.91**	**0.92**
	2	0.07	0.91	**0.84**	**0.86**	**0.80**	**0.89**	**0.90**
	4	0.13	0.83	**0.79**	0.86	**0.75**	0.88	0.88
	8	0.22	0.79	**0.76**	0.86	**0.73**	0.85	0.85
	12	0.27	0.82	**0.79**	0.88	**0.76**	0.86	0.86
	16	0.30	0.85	**0.81**	0.90	**0.78**	0.87	0.87
10	1	0.04	1.00	**0.92**	**0.85**	**0.88**	**0.91**	**0.92**
	2	0.07	0.91	**0.85**	**0.83**	**0.80**	**0.88**	**0.88**
	4	0.13	0.83	**0.78**	**0.82**	**0.75**	0.85	0.85
	8	0.22	0.79	**0.74**	0.82	**0.72**	0.83	0.82
	12	0.27	0.82	**0.77**	0.86	**0.74**	0.84	0.84
	16	0.30	0.85	**0.79**	0.88	**0.77**	0.85	0.85
20	1	0.04	1.00	**0.93**	**0.85**	**0.89**	**0.90**	**0.92**
	2	0.07	0.91	**0.85**	**0.81**	**0.81**	**0.86**	**0.87**
	4	0.13	0.83	**0.77**	**0.79**	**0.75**	0.83	0.83
	8	0.22	0.79	**0.72**	**0.79**	**0.71**	0.80	0.80
	12	0.27	0.82	**0.75**	0.83	**0.73**	0.82	0.82
	16	0.30	0.85	**0.78**	0.86	**0.76**	**0.84**	**0.83**
∞	1	0.04	1.00	**0.88**	**0.95**	**0.87**	8.24	1.02
	2	0.07	0.91	**0.70**	**0.79**	**0.70**	8.17	0.92
	4	0.13	0.83	**0.58**	**0.67**	**0.58**	4.47	**0.82**
	8	0.22	0.79	**0.55**	**0.62**	**0.56**	2.08	**0.77**
	12	0.27	0.82	**0.62**	**0.66**	**0.62**	2.35	**0.79**
	16	0.30	0.85	**0.69**	**0.71**	**0.70**	2.16	**0.82**

See notes to table 2 in body of paper.

Table A11: Root MSFEs for Hours for 180 Quarter Estimation Window

λ	Horizon	Unrestricted RMSFE	VAR-MINN	RBC	HABIT	ISHOCK	STICKY	STICKY-ACC
0.25	1	0.01	1.19	1.00	1.00	***0.99***	1.09	1.06
	2	0.01	1.22	**0.99**	**1.00**	***0.96***	1.11	1.06
	4	0.02	1.16	**0.95**	**0.99**	***0.92***	1.09	1.04
	8	0.04	1.10	**0.93**	**0.98**	***0.90***	1.05	1.02
	12	0.05	1.12	**0.94**	**1.00**	***0.91***	1.05	1.02
	16	0.06	1.12	**0.96**	1.00	***0.93***	1.04	1.02
0.5	1	0.01	1.19	1.00	1.01	***0.98***	1.12	1.08
	2	0.01	1.22	**0.98**	**1.00**	***0.94***	1.13	1.08
	4	0.02	1.16	**0.93**	**0.99**	***0.89***	1.11	1.06
	8	0.04	1.10	**0.91**	**0.98**	***0.87***	1.06	1.03
	12	0.05	1.12	**0.92**	**0.99**	***0.88***	1.06	1.03
	16	0.06	1.12	**0.94**	**1.00**	***0.90***	1.05	1.02
1	1	0.01	1.19	1.01	1.02	***0.97***	1.15	1.10
	2	0.01	1.22	**0.99**	1.01	***0.93***	1.16	1.10
	4	0.02	1.16	**0.93**	**0.99**	***0.86***	1.13	1.07
	8	0.04	1.10	**0.89**	**0.98**	***0.83***	1.08	1.03
	12	0.05	1.12	**0.90**	**0.99**	***0.85***	1.07	1.03
	16	0.06	1.12	**0.91**	**0.99**	***0.86***	1.05	1.01
5	1	0.01	1.19	1.05	1.05	***0.98***	1.20	1.15
	2	0.01	1.22	1.03	1.04	***0.93***	1.22	1.16
	4	0.02	1.16	**0.94**	1.02	***0.84***	1.18	1.11
	8	0.04	1.10	**0.87**	**0.99**	***0.79***	1.10	1.05
	12	0.05	1.12	**0.87**	**1.00**	***0.79***	1.08	1.03
	16	0.06	1.12	**0.87**	**0.98**	***0.80***	1.05	1.01
10	1	0.01	1.19	1.07	1.06	***0.99***	1.22	1.18
	2	0.01	1.22	1.06	1.06	***0.95***	1.24	1.19
	4	0.02	1.16	**0.96**	1.03	***0.85***	1.19	1.14
	8	0.04	1.10	**0.88**	1.00	***0.79***	1.12	1.06
	12	0.05	1.12	**0.87**	1.00	***0.78***	1.09	1.04
	16	0.06	1.12	**0.86**	**0.98**	***0.78***	1.05	1.01
20	1	0.01	1.19	1.10	1.09	1.02	1.23	1.21
	2	0.01	1.22	1.09	1.09	***0.98***	1.26	1.22
	4	0.02	1.16	**0.99**	1.05	***0.88***	1.21	1.16
	8	0.04	1.10	**0.89**	1.01	***0.80***	1.13	1.07
	12	0.05	1.12	**0.87**	1.01	***0.79***	1.10	1.05
	16	0.06	1.12	**0.86**	**0.99**	***0.78***	1.05	1.02
∞	1	*0.01*	1.19	1.35	1.51	1.34	23.88	1.40
	2	*0.01*	1.22	1.42	1.62	1.41	26.08	1.50
	4	*0.02*	1.16	1.30	1.45	1.30	7.24	1.40
	8	*0.04*	1.10	1.11	1.20	1.11	1.64	1.22
	12	*0.05*	1.12	1.06	1.13	1.06	3.83	1.17
	16	*0.06*	1.12	1.02	1.07	1.02	2.68	1.11

See notes to table 2 in body of paper.

Table A12: Root MSFEs for Consumption for 180 Quarter Estimation Window

λ	Horizon	Unrestricted RMSFE	VAR-MINN	RBC	HABIT	ISHOCK	STICKY	STICKY-ACC
0.25	1	0.01	1.04	**0.98**	1.00	***0.97***	**1.00**	1.00
	2	0.01	0.95	0.96	0.99	***0.94***	1.00	1.01
	4	0.03	*0.90*	0.96	0.99	0.94	1.00	1.00
	8	0.06	*0.87*	0.96	0.98	0.95	0.98	0.99
	12	0.09	*0.89*	0.97	0.98	0.96	0.97	0.98
	16	0.12	*0.90*	0.98	0.98	0.97	0.97	0.97
0.5	1	0.01	1.04	**0.98**	**1.00**	***0.96***	**0.99**	**1.00**
	2	0.01	0.95	**0.95**	0.98	***0.93***	0.99	1.00
	4	0.03	*0.90*	0.94	0.98	0.93	0.98	0.99
	8	0.06	*0.87*	0.94	0.97	0.94	0.96	0.97
	12	0.09	*0.89*	0.96	0.97	0.96	0.95	0.96
	16	0.12	*0.90*	0.97	0.97	0.97	0.95	0.96
1	1	0.01	1.04	**0.97**	**1.00**	***0.96***	**0.98**	**0.99**
	2	0.01	0.95	**0.94**	0.97	***0.93***	0.96	0.98
	4	0.03	*0.90*	0.92	0.96	0.92	0.95	0.97
	8	0.06	*0.87*	0.93	0.95	0.93	0.94	0.95
	12	0.09	*0.89*	0.94	0.95	0.95	0.93	0.94
	16	0.12	*0.90*	0.95	0.96	0.97	0.93	0.94
5	1	0.01	1.04	**0.97**	**0.99**	**0.97**	***0.96***	**0.98**
	2	0.01	0.95	***0.91***	**0.94**	**0.93**	**0.91**	**0.93**
	4	0.03	0.90	***0.88***	0.93	0.91	**0.89**	0.91
	8	0.06	*0.87*	0.88	0.91	0.91	0.88	0.89
	12	0.09	*0.89*	0.90	0.92	0.93	0.88	0.89
	16	0.12	*0.90*	0.92	0.93	0.95	0.88	0.89
10	1	0.01	1.04	**0.97**	**0.98**	**0.98**	***0.95***	**0.97**
	2	0.01	0.95	**0.90**	**0.93**	**0.93**	***0.89***	**0.92**
	4	0.03	0.90	***0.87***	0.91	0.90	**0.87**	**0.89**
	8	0.06	0.87	**0.86**	0.90	0.90	***0.86***	**0.87**
	12	0.09	0.89	**0.88**	0.91	0.92	***0.86***	**0.87**
	16	0.12	0.90	**0.90**	0.92	0.94	***0.87***	**0.87**
20	1	0.01	1.04	**0.96**	**0.98**	**0.98**	***0.95***	**0.97**
	2	0.01	0.95	**0.88**	**0.91**	**0.92**	***0.87***	**0.90**
	4	0.03	0.90	**0.84**	**0.89**	**0.89**	***0.84***	**0.86**
	8	0.06	0.87	**0.84**	0.88	0.88	***0.83***	**0.84**
	12	0.09	0.89	**0.86**	0.89	0.90	***0.84***	**0.84**
	16	0.12	0.90	**0.88**	0.91	0.92	***0.85***	**0.85**
∞	1	0.01	1.04	***0.94***	**0.98**	**0.95**	1.02	**0.96**
	2	0.01	**0.95**	**0.60**	**0.72**	***0.59***	**0.71**	**0.76**
	4	0.03	**0.90**	**0.55**	**0.67**	***0.53***	**0.75**	**0.71**
	8	0.06	**0.87**	**0.59**	**0.68**	***0.57***	**0.77**	**0.70**
	12	0.09	**0.89**	**0.64**	**0.72**	***0.62***	**0.77**	**0.73**
	16	0.12	**0.90**	**0.69**	**0.75**	***0.67***	**0.83**	**0.76**

See notes to table 2 in body of paper.

2 Writing the Models in State-Space Form

2.1 Models 1 and 3: RBC Model with Additional Shocks

2.1.1 The Log-Linearized Model

Let lower case variables denote the percent deviation of detrended variables from their steady state values and $\kappa = \frac{\eta}{\beta} - 1 + \delta$ and $\lambda = \eta - 1 + \delta$. Then the log-linearized equilibrium conditions for the detrended model are

$$y_t = \left[1 - \frac{g}{y} - \theta\frac{\lambda}{\kappa}\right] c_t + \theta\frac{\lambda}{\kappa} i_t + \frac{g}{y} g_t \tag{1}$$

$$y_t = A_t + \theta k_t + (1-\theta) H_t \tag{2}$$

$$\eta k_{t+1} = (1-\delta) k_t + \lambda i_t + \lambda V_t \tag{3}$$

$$y_t = c_t + H_t \tag{4}$$

$$0 = \frac{\eta}{\beta} c_t - \frac{\eta}{\beta} E_t c_{t+1} + \kappa E_t y_{t+1} - \kappa E_t k_{t+1} + \left(\frac{\eta}{\beta} - (1-\delta)\rho_V\right) V_t \tag{5}$$

$$A_t = \rho_A A_{t-1} + \varepsilon_t^A \tag{6}$$

$$g_t = \rho_g g_{t-1} + \varepsilon_t^g \tag{7}$$

$$V_t = \rho_V V_{t-1} + \varepsilon_t^V. \tag{8}$$

2.1.2 Putting the Model in Blanchard-Kahn Form

Let $f_t^0 = [y_t, i_t, H_t]'$, $s_t^0 = [k_t, c_t]'$, $z_t^0 = [A_t, g_t, V_t]'$. Then we can write (1), (2), and (4) as

$$f_t^0 = A^{-1} B s_t^0 + A^{-1} C z_t^0 \tag{9}$$

where

$$A = \begin{bmatrix} 1 & -\theta\frac{\lambda}{\kappa} & 0 \\ 1 & 0 & \theta - 1 \\ 1 & 0 & -1 \end{bmatrix},$$

$$B = \begin{bmatrix} 0 & 1 - \frac{g}{y} - \theta\frac{\lambda}{\kappa} \\ \theta & 0 \\ 0 & 1 \end{bmatrix},$$

and

$$C = \begin{bmatrix} 0 & \frac{g}{y} & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

We can also write (3) and (5) as

$$DE_t s_{t+1}^0 + FE_t f_{t+1}^0 = Gs_t^0 + Hf_t^0 + Jz_t^0 \tag{10}$$

where

$$\begin{aligned} D &= \begin{bmatrix} \eta & 0 \\ \kappa & \frac{\eta}{\beta} \end{bmatrix}, \\ F &= \begin{bmatrix} 0 & 0 & 0 \\ -\kappa & 0 & 0 \end{bmatrix}, \\ G &= \begin{bmatrix} 1-\delta & 0 \\ 0 & \frac{\eta}{\beta} \end{bmatrix}, \\ H &= \begin{bmatrix} 0 & \lambda & 0 \\ 0 & 0 & 0 \end{bmatrix}, \end{aligned}$$

and

$$J = \begin{bmatrix} 0 & 0 & \lambda \\ 0 & 0 & \frac{\eta}{\beta} - (1-\delta)\,\rho_V \end{bmatrix}.$$

Now plug (9) into (10) to get

$$E_t s_{t+1}^0 = Ms_t^0 + Nz_t^0 \tag{11}$$

where

$$\begin{aligned} M &= \left(D + FA^{-1}B\right)^{-1}\left(G + HA^{-1}B\right), \\ N &= \left(D + FA^{-1}B\right)^{-1}\left(HA^{-1}C + J - L\right), \\ L &= \begin{bmatrix} K_{11}\rho_A & K_{12}\rho_g & K_{13}\rho_V \\ K_{21}\rho_A & K_{22}\rho_g & K_{23}\rho_V \end{bmatrix}, \end{aligned}$$

and $K = FA^{-1}C$.

We can now decompose M into its Jordan canonical form by writing $M = P^{-1}QP$ where the columns of P^{-1} are the eigenvectors of M and the diagonal entries of Q are its eigenvalues in ascending order. Premultiply (11) by P and let $R = PN$ and $\lambda_{iM} = Q_{ii}$ we have

$$E_t s^1_{1,t+1} = \lambda_{iM} s^1_{1,t+1} + R_{11} A_t + R_{12} g_t + R_{13} V_t \tag{12}$$

$$E_t s^1_{2,t+1} = \lambda_{iM} s^1_{2,t+1} + R_{21} A_t + R_{22} g_t + R_{23} V_t \tag{13}$$

where

$$s^1_{1,t} = P_{11} k_t + P_{12} c_t \tag{14}$$

$$s^1_{2,t} = P_{21} k_t + P_{22} c_t. \tag{15}$$

Since $|\lambda_{2M}| < 1$, we have

$$s^1_{2,t} = \frac{R_{21}}{\rho_A - \lambda_{2M}} A_t + \frac{R_{22}}{\rho_g - \lambda_{2M}} g_t + \frac{R_{23}}{\rho_V - \lambda_{2M}} V_t. \tag{16}$$

Using this in (15), we have

$$c_t = S_1 k_t + S_2 A_t + S_3 g_t + S_4 V_t \tag{17}$$

where

$$\begin{aligned} S_1 &= -\frac{P_{21}}{P_{22}}, \\ S_2 &= \frac{R_{21}}{P_{22}\left(\rho_A - \lambda_{2M}\right)}, \\ S_3 &= \frac{R_{22}}{P_{22}\left(\rho_g - \lambda_{2M}\right)}, \\ S_4 &= \frac{R_{23}}{P_{22}\left(\rho_V - \lambda_{2M}\right)}. \end{aligned} \tag{18}$$

Combining (17) and (14) and substituting into (12), we get

$$k_{t+1} = S_5 k_t + S_6 A_t + S_7 g_t + S_8 V_t$$

where

$$
\begin{aligned}
S_5 &= \lambda_{1M}, \\
S_6 &= \frac{P_{12}S_2\left(\lambda_{1M}-\rho_A\right)+R_{11}}{P_{11}+P_{12}S_1}, \\
S_7 &= \frac{P_{12}S_3\left(\lambda_{1M}-\rho_g\right)+R_{12}}{P_{11}+P_{12}S_1}, \\
S_8 &= \frac{P_{12}S_4\left(\lambda_{1M}-\rho_V\right)+R_{13}}{P_{11}+P_{12}S_1}.
\end{aligned}
$$

Substituting (17) into (9) we have

$$f_t^0 = T_1 k_t + T_2 z_t^0$$

where

$$
\begin{aligned}
T_1 &= A^{-1}B\begin{bmatrix} 1 \\ S_1 \end{bmatrix} \\
T_2 &= A^{-1}C + A^{-1}B\begin{bmatrix} 0 & 0 & 0 \\ S_2 & S_3 & S_4 \end{bmatrix}.
\end{aligned}
$$

Now let $s_t = [k_t, A_t, g_t, V_t]'$ and $\varepsilon_t = \left[\varepsilon_t^A, \varepsilon_t^g, \varepsilon_t^V\right]'$. Then we can write the state transition equation as

$$s_{t+1} = \Pi s_t + W\varepsilon_t$$

where

$$
\Pi = \begin{bmatrix} S_5 & S_6 & S_7 & S_8 \\ 0 & \rho_A & 0 & 0 \\ 0 & 0 & \rho_G & 0 \\ 0 & 0 & 0 & \rho_V \end{bmatrix}
$$

and

$$
W = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.
$$

Finally, let $f_t = [y_t, i_t, H_t, c_t]'$ and add a measurement error to the equation for y_t to ensure we have as many shocks in the system as observed variables. We can then write the observation equation as

$$f_t = U s_t + v_t$$

where

$$U = \begin{bmatrix} T_1 & T_2 & & \\ S_1 & S_2 & S_3 & S_4 \end{bmatrix}$$

and $v_t = [\varepsilon_t^y, 0, 0, 0]'$.

2.2 Model 2: Habit Formation and Capital Adjustment Costs

2.2.1 Equilibrium Conditions

For the detrended model, we have

$$\begin{aligned}
\frac{1}{c_t - \left(\frac{\nu}{\eta}\right) c_{t-1}} - E_t \left[\frac{\beta\nu/\eta}{c_{t+1} - (\nu/\eta)\, c_t}\right] &= \lambda_t \\
\lambda_t (1-\theta) A_t k_t^{\theta} H_t^{-\theta} &= \gamma \\
\beta E_t \lambda_{t+1} \left[\begin{array}{c} \theta A_{t+1} k_{t+1}^{\theta-1} H_{t+1}^{1-\theta} + (1-\delta) \frac{k_{t+1}}{V_{t+1}} \\ +q \frac{\eta^2 (k_{t+2} - k_{t+1})}{k_{t+1}} + q\eta^2 \frac{(k_{t+2}-k_{t+1})^2}{2k_{t+1}^2} \end{array}\right] &= \lambda_t \left(\frac{\eta}{V_t} + q\eta^2 \frac{k_{t+1} - k_t}{k_t}\right) \\
c_t + g_t + \eta \frac{k_{t+1}}{V_t} + (1-\delta) \frac{k_{t+1}}{V_t} + q\eta^2 \frac{(k_{t+1} - k_t)^2}{2k_t} &= A_t k_t^{\theta} H_t^{1-\theta}
\end{aligned}$$

2.2.2 Steady State

In the steady state we have

$$\begin{aligned}
\frac{k}{y} &= \frac{\theta\beta}{\eta - \beta(1-\delta)} \\
s_i &= \frac{i}{y} = \frac{\theta(\eta - 1 + \delta)}{\eta/\beta - 1 + \delta}. \\
s_c &= 1 - \frac{g}{y} - s_i
\end{aligned}$$

2.2.3 The Log-Linearized Model

Define

$$\begin{aligned}
\psi &= \frac{\eta - \beta(1-\delta)}{\eta} \\
a_1 &= \frac{\beta\nu/\eta}{1 - \nu/\eta} \\
a_2 &= -\frac{1 + \beta(\nu/\eta)^2}{1 - \nu/\eta} \\
a_3 &= \frac{\nu/\eta}{1 - \nu/\eta} \\
a_4 &= 1 - \frac{\beta\nu}{\eta}.
\end{aligned}$$

Then the log linearized equations can be expressed as

$$
\begin{aligned}
a_1 c_{t+1} + a_2 c_t + a_3 c_{t-1} &= a_4 \lambda_t && (19)\\
\theta H_t - \theta k_t - A_t &= \lambda_t && (20)\\
\left[\begin{array}{c} E_t\lambda_{t+1} + \psi E_t A_{t+1} + \psi\left(1-\theta\right) E_t H_{t+1} - \psi\left(1-\theta\right) E_t k_{t+1} \\ -\left(1-\delta\right)\frac{\beta}{\eta} E_t V_{t+1} + \beta q\eta E_t k_{t+2} - \beta q \eta k_{t+1} \end{array}\right] &= \lambda_t - V_t + q\eta k_{t+1} - q\eta k_t && (21)\\
\frac{g}{y} g_t + s_c c_t + \eta \frac{k}{y} k_{t+1} - \left(1-\delta\right)\frac{k}{y} k_t + \frac{k}{y}\left(1-\delta-\eta\right) V_t &= A_t + \theta k_t + \left(1-\theta\right) H_t. && (22)
\end{aligned}
$$

Now make the substitution $\hat{h}_t = \hat{c}_{t-1}$ and use (22) to write H_t as

$$
\begin{aligned}
H_t &= \frac{s_c}{1-\theta} c_t + \eta \frac{k/y}{1-\theta} k_{t+1} - \frac{\left(1-\delta\right) k/y + \theta}{1-\theta} k_t + k/y \frac{1-\delta-\eta}{1-\theta} V_t - \frac{1}{1-\theta} A_t + \frac{g/y}{1-\theta} g_t && (23)\\
&= a_5 c_t + a_6 k_{t+1} + a_7 k_t + a_8 A_t + a_9 g_t + a_{10} V_t\\
a_5 &= \frac{s_c}{1-\theta}\\
a_6 &= \eta \frac{k/y}{1-\theta}\\
a_7 &= -\frac{\left(1-\delta\right) k/y + \theta}{1-\theta}\\
a_8 &= -\frac{1}{1-\theta}\\
a_9 &= \frac{g/y}{1-\theta}\\
a_{10} &= k/y \frac{1-\delta-\eta}{1-\theta}.
\end{aligned}
$$

Now use (20) to write λ_t as

$$
\begin{aligned}
\lambda_t &= \theta a_5 c_t + \theta a_6 k_{t+1} + \left(\theta a_7 - \theta\right) k_t + \left(\theta a_8 - 1\right) A_t + \theta a_9 g_t + \theta a_{10} V_t\\
&= a_{11} c_t + a_{12} k_{t+1} + a_{13} k_t + a_{14} A_t + a_{15} g_t + a_{16} V_t\\
a_{11} &= \theta a_5\\
a_{12} &= \theta a_6\\
a_{13} &= \theta\left(a_7 - 1\right)\\
a_{14} &= \theta a_8 - 1\\
a_{15} &= \theta a_9\\
a_{16} &= \theta a_{10}.
\end{aligned}
$$

2.2.4 Putting the Model in Blanchard-Kahn Form

To write this as a first order difference system let $P_t^1 = h_{t+1}$ and $P_t^2 = k_{t+1}$. Now write (19) and (21) exlcusively in terms of h, k, P^1, P^2, A, g, and V:

Now and let $s_t^0 = \left[h_t, k_t, P_t^1, P_t^2\right]'$ and $z_t^0 = [A_t, g_t, V_t]$ Then we can write the system as

$$\begin{aligned}
b_1 E_t P_{t+1}^1 + b_2 h_{t+1} &= -b_3 h_t + b_4 P_t^2 + b_5 k_t + b_6 A_t + b_7 g_t + b_8 V_t, \\
b_1 &= a_1 \\
b_2 &= a_2 - a_4 a_{11} \\
b_3 &= a_3 \\
b_4 &= a_4 a_{12} \\
b_5 &= a_4 a_{13} \\
b_6 &= a_4 a_{14} \\
b_7 &= a_4 a_{15} \\
b_8 &= a_4 a_{16}
\end{aligned}$$

$$\begin{aligned}
b_9 E_t P_{t+1}^1 + b_{10} E_t P_{t+1}^2 + b_{11} k_{t+1} &= b_{12} P_t^1 + b_{13} k_t + b_{14} A_t + b_{15} g_t + b_{16} V_t, \\
b_9 &= a_{11} + \psi (1-\theta) a_5 \\
b_{10} &= a_{12} + \psi (1-\theta) a_6 + \beta q \eta \\
b_{11} &= a_{13} - \psi (1-\theta) + \psi (1-\theta) a_7 - \beta q \eta - q \eta - a_{12} \\
b_{12} &= a_{11} \\
b_{13} &= a_{13} - q\eta \\
b_{14} &= a_{14} (1-\rho_A) - \psi \rho_A - \psi (1-\theta) \rho_A a_8 \\
b_{15} &= a_{15} \left(1-\rho_g\right) - \psi (1-\theta) \rho_G a_9 \\
b_{16} &= a_{16} (1-\rho_V) - 1 - \psi (1-\theta) \rho_V a_{10} + (1-\delta) \frac{\beta}{\eta} \rho_V
\end{aligned}$$

$$\begin{aligned}
h_{t+1} &= P_t^1 \\
k_{t+1} &= P_t^2.
\end{aligned}$$

where h_t and k_t are predetermined at t and P_t^1 and P_t^2 are not.

Letting $s_t^0 = \left[h_t, k_t, P_t^1, P_t^2\right]'$ and $z_t^0 = [A_t, g_t, V_t]'$ we can write this as

$$E_t s_{t+1}^0 = A^{-1} B s_t^0 + A^{-1} C z_t^0 \tag{24}$$

where

$$\begin{aligned}
A &= \begin{bmatrix} b_2 & 0 & b_1 & 0 \\ 0 & b_{11} & b_9 & b_{10} \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}, \\
B &= \begin{bmatrix} -b_3 & b_5 & 0 & b_4 \\ 0 & b_{13} & b_{12} & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \\
C &= \begin{bmatrix} b_6 & b_7 & b_8 \\ b_{14} & b_{15} & b_{16} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.
\end{aligned}$$

Let $D = A^{-1}B$ and write D in Jordan canonical form, $D = G^{-1}HG$ where G^{-1} contains the eigenvectors of D and H has the eigenvalues of D in ascending order along the diagonal.

Multiplying (24) by G and letting $J = GA^{-1}C$, denoting G_{ij} the ijth element of G, J_{ij} the ijth element of J, and λ_{iH} the iith element of H we can define

$$
\begin{aligned}
s^1_{1,t} &= G_{11}h_t + G_{12}k_t + G_{13}P^1_t + G_{14}P^2_t && (25)\\
s^1_{2,t} &= G_{21}h_t + G_{22}k_t + G_{23}P^1_t + G_{24}P^2_t && (26)\\
s^1_{3,t} &= G_{31}h_t + G_{32}k_t + G_{33}P^1_t + G_{34}P^2_t && (27)\\
s^1_{4,t} &= G_{41}h_t + G_{42}k_t + G_{43}P^1_t + G_{44}P^2_t && (28)
\end{aligned}
$$

and write

$$
\begin{aligned}
E_t s^1_{1,t+1} &= \lambda_{1H} s^1_{1,t} + J_{11}A_t + J_{12}g_t + J_{13}V_t && (29)\\
E_t s^1_{2,t+1} &= \lambda_{2H} s^1_{2,t} + J_{21}A_t + J_{22}g_t + J_{23}V_t && (30)\\
E_t s^1_{3,t+1} &= \lambda_{3H} s^1_{1,t} + J_{31}A_t + J_{32}g_t + J_{33}V_t && (31)\\
E_t s^1_{4,t+1} &= \lambda_{4H} s^1_{1,t} + J_{41}A_t + J_{42}g_t + J_{43}V_t && (32)
\end{aligned}
$$

Since $|\lambda_{3H}|, |\lambda_{4H}| < 1$, we can write

$$
\begin{aligned}
s^1_{3,t} &= \frac{J_{31}}{\rho_A - \lambda_{3H}}A_t + \frac{J_{32}}{\rho_g - \lambda_{3H}}g_t + \frac{J_{33}}{\rho_V - \lambda_{3H}} && (33)\\
s^1_{4,t} &= \frac{J_{41}}{\rho_A - \lambda_{4H}}A_t + \frac{J_{42}}{\rho g - \lambda_{4H}}g_t + \frac{J_{43}}{\rho_V - \lambda_{4H}}. && (34)
\end{aligned}
$$

Then we can combine (27), (28), (33), and (34) to write

$$
\begin{aligned}
P^1_t &= d_6h_t + d_7k_t + d_8A_t + d_9g_t + d_{10}V_t && (35)\\
P^2_t &= d_1h_t + d_2k_t + d_3A_t + d_4g_t + d_5V_t && (36)
\end{aligned}
$$

where

$$
\begin{aligned}
d_1 &= \left(1-\frac{G_{43}}{G_{44}}\frac{G_{34}}{G_{33}}\right)^{-1}\left(\frac{G_{43}G_{31}}{G_{44}G_{33}}-\frac{G_{41}}{G_{44}}\right)\\
d_2 &= \left(1-\frac{G_{43}}{G_{44}}\frac{G_{34}}{G_{33}}\right)^{-1}\left(\frac{G_{43}G_{32}}{G_{44}G_{33}}-\frac{G_{42}}{G_{44}}\right)\\
d_3 &= \left(1-\frac{G_{43}}{G_{44}}\frac{G_{34}}{G_{33}}\right)^{-1}\left[\frac{J_{41}}{G_{44}\left(\rho_A-\lambda_{4H}\right)}-\frac{G_{43}}{G_{44}}\frac{J_{31}}{G_{33}\left(\rho_A-\lambda_{3H}\right)}\right]\\
d_4 &= \left(1-\frac{G_{43}}{G_{44}}\frac{G_{34}}{G_{33}}\right)^{-1}\left[\frac{J_{42}}{G_{44}\left(\rho_g-\lambda_{4H}\right)}-\frac{G_{43}}{G_{44}}\frac{J_{32}}{G_{33}\left(\rho_g-\lambda_{3H}\right)}\right]\\
d_5 &= \left(1-\frac{G_{43}}{G_{44}}\frac{G_{34}}{G_{33}}\right)^{-1}\left[\frac{J_{43}}{G_{44}\left(\rho_V-\lambda_{4H}\right)}-\frac{G_{43}}{G_{44}}\frac{J_{33}}{G_{33}\left(\rho_V-\lambda_{3H}\right)}\right]\\
d_6 &= -\left[\frac{G_{31}}{G_{33}}+\frac{G_{43}}{G_{44}}d_1\right]\\
d_7 &= -\left[\frac{G_{32}}{G_{33}}+\frac{G_{43}}{G_{44}}d_2\right]\\
d_8 &= \frac{J_{31}}{\left(\rho_A-\lambda_{3H}\right)G_{33}}-\frac{G_{43}}{G_{44}}d_3\\
d_9 &= \frac{J_{32}}{\left(\rho_g-\lambda_{3H}\right)G_{33}}-\frac{G_{43}}{G_{44}}d_4\\
d_{10} &= \frac{J_{33}}{\left(\rho_V-\lambda_{3H}\right)G_{33}}-\frac{G_{43}}{G_{44}}d_5.
\end{aligned}
$$

Now substituting (35) and (36) into (25) and (26) yields

$$
\begin{aligned}
j_1h_{t+1}+j_2k_{t+1} &= j_3h_t+j_4k_t+j_5A_t+j_6g_t+j_7V_t\\
j_8h_{t+1}+j_9k_{t+1} &= j_{10}h_t+j_{11}k_t+j_{12}A_t+j_{13}g_t+j_{14}V_t
\end{aligned}
$$

where

$$
\begin{aligned}
j_1 &= G_{11}+G_{13}d_6+G_{14}d_1\\
j_2 &= G_{12}+G_{13}d_7+G_{14}d_2\\
j_3 &= \lambda_{1H}j_1\\
j_4 &= \lambda_{1H}j_2\\
j_5 &= \left(\lambda_{1H}-\rho_A\right)\left(G_{13}d_8+G_{14}d_3\right)+J_{11}\\
j_6 &= \left(\lambda_{1H}-\rho_G\right)\left(G_{13}d_9+G_{14}d_4\right)+J_{12}\\
j_7 &= \left(\lambda_{1H}-\rho_V\right)\left(G_{13}d_{10}+G_{14}d_5\right)+J_{13}\\
j_8 &= G_{21}+G_{23}d_6+G_{24}d_1\\
j_9 &= G_{22}+G_{23}d_7+G_{24}d_2\\
j_{10} &= \lambda_{2H}j_8\\
j_{11} &= \lambda_{2H}j_9\\
j_{12} &= \left(\lambda_{2H}-\rho_A\right)\left(G_{23}d_8+G_{24}d_3\right)+J_{21}\\
j_{13} &= \left(\lambda_{2H}-\rho_G\right)\left(G_{23}d_9+G_{24}d_4\right)+J_{22}\\
j_{14} &= \left(\lambda_{2H}-\rho_G\right)\left(G_{23}d_{10}+G_{24}d_5\right)+J_{12}.
\end{aligned}
$$

Let $s_t=[h_t,k_t,A_t,g_t,V_t]'$ and $\varepsilon_t=\left[\varepsilon_t^A,\varepsilon_t^g,\varepsilon_t^V\right]'$. Then the state transition equation is

$$
s_{t+1}=\Pi s_t+W\varepsilon_t
$$

where

$$\Pi = \begin{bmatrix} j_1 & j_2 & 0 & 0 & 0 \\ j_8 & j_9 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}^{-1} \begin{bmatrix} j_3 & j_4 & j_5 & j_6 & j_7 \\ j_{10} & j_{11} & j_{12} & j_{13} & j_{14} \\ 0 & 0 & \rho_A & 0 & 0 \\ 0 & 0 & 0 & \rho_G & 0 \\ 0 & 0 & 0 & 0 & \rho_V \end{bmatrix}$$

$$W = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

Then letting $f_t = [y_t, i_t, H_t, c_t]'$ and $v_t = [\varepsilon_t^y, 0, 0, 0]'$ we have

$$f_t = U s_t + v_t$$

where

$$U = \begin{bmatrix} m_6 & m_7 & m_8 & m_9 & m_{10} \\ m_{11} & m_{12} & m_{13} & m_{14} & m_{15} \\ m_1 & m_2 & m_3 & m_4 & m_5 \\ \Pi_{11} & \Pi_{12} & \Pi_{13} & \Pi_{14} & \Pi_{15} \end{bmatrix},$$

$$\begin{aligned}
m_1 &= a_5\Pi_{11} + a_6\Pi_{21} \\
m_2 &= a_5\Pi_{12} + a_6\Pi_{22} + a_7 \\
m_3 &= a_5\Pi_{13} + a_6\Pi_{23} + a_8 \\
m_4 &= a_5\Pi_{14} + a_6\Pi_{24} + a_9 \\
m_5 &= a_5\Pi_{15} + a_6\Pi_{25} + a_{10} \\
m_6 &= m_1(1-\theta) \\
m_7 &= \theta + m_2(1-\theta) \\
m_8 &= 1 + m_3(1-\theta) \\
m_9 &= m_4(1-\theta) \\
m_{10} &= m_5(1-\theta) \\
m_{11} &= \frac{\eta}{\eta - 1 + \delta}\Pi_{21} \\
m_{12} &= \frac{\eta}{\eta - 1 + \delta}\Pi_{22} - \frac{1-\delta}{\eta - 1 + \delta} \\
m_{13} &= \frac{\eta}{\eta - 1 + \delta}\Pi_{23} \\
m_{14} &= \frac{\eta}{\eta - 1 + \delta}\Pi_{24} \\
m_{15} &= \frac{\eta}{\eta - 1 + \delta}\Pi_{25}.
\end{aligned}$$

2.3 A Sticky Price Model with an Unaccommodating Monetary Authority

2.3.1 Final Good Producers

Final goods produces have access to the technology

$$Y_t = \left[\int_0^1 Y_{j,t}^{\frac{\xi-1}{\xi}} dj \right]^{\frac{\xi}{\xi-1}}$$

Profit maximization leads to

$$Y_{j,t}^d = \left(\frac{P_t}{P_{j,t}} \right)^{\xi} Y_t$$

and substituting this into the aggregate price level yields

$$P_t = \left[\int_0^1 P_{j,t}^{1-\xi} dj \right]^{\frac{1}{1-\xi}} .$$

2.3.2 Households

The household's first order conditions are

$$\begin{aligned}
\frac{W_t}{C_t} &= \gamma \\
\frac{1}{V_t C_t} &= \beta E_t \left[\frac{1}{C_{t+1}} \left(r_{t+1} + \frac{1-\delta}{V_{t+1}} + \eta q \frac{K_{t+1} - \eta K_t}{K_t} + q \frac{(K_{t+1} - \eta K_t)^2}{2K_t^2} \right) \right] \\
\frac{1}{C_t} &= \omega \frac{P_t}{M_t^D} + \beta E_t \left(\frac{P_t}{C_{t+1} P_{t+1}} \right)
\end{aligned}$$

2.3.3 Intermediate Goods Firms

We have

$$MC_t = A_t^{-1} r_t^{\theta} \left(\eta^t \right)^{\theta - 1} W_t^{1-\theta} \left(\frac{1}{1-\theta} \right)^{1-\theta} \theta^{-\theta}.$$

Cost minimization implies

$$\frac{W_t}{r_t} = \left(\frac{1-\theta}{\theta} \right) \frac{K_{j,t}}{H_{j,t}}$$

and the first order condition for optimal price-setting implies that firms that are able to reoptimize all set

$$P_{j,t}^* = \frac{\xi E_t \sum_{k=0}^{\infty} (\alpha\beta)^k \frac{Y_{t+k}}{C_{t+k}} P_{t+k}^{\xi} MC_{t+k}}{(\xi - 1) E_t \sum_{k=0}^{\infty} (\alpha\beta)^k \frac{Y_{t+k}}{C_{t+k}} P_{t+k}^{\xi-1}}.$$

2.3.4 Aggregate Price Level

By well-known results in the literature, the aggregate price level evolves as

$$P_t^{1-\xi} = (1-\alpha) P_{j,t}^{*1-\xi} + \alpha P_{t-1}^{1-\xi}.$$

2.3.5 The Monetary Authority

The monetary authority follows

$$M_{t+1}^S = \mu M_t^S$$

and remits all seignorage revenue to the household as transfers, i.e.,

$$T_t = (\mu - 1) M_{t-1}^S.$$

2.3.6 Resource Constraint

The resource constraint is, to a first order approximation (see, e.g., Yun (1996) and CEE (2005))

$$\begin{aligned} Y_t &= A_t K_{j,t}^{\theta} \left(\eta^t H_{j,t}\right)^{1-\theta} = C_t + I_t + G_t + q\frac{\left(K_t - \eta K_{t-1}\right)^2}{2K_{t-1}^2} \\ I_t &= \frac{K_t - (1-\delta) K_{t-1}}{V_t} \end{aligned}$$

2.3.7 Equilibrium

In equilibrium, $K_{j,t} = K_{t-1}$, $H_{j,t} = H_t$, and $M_t^D = M_t^S = M_t$. The equilibrium in terms of detrended variables is then defined by

$$\frac{w_t}{c_t} = \gamma \tag{37}$$

$$\frac{1}{c_t}\left[\frac{1}{V_t} + \eta q\left(\frac{k_t}{k_{t-1}} - 1\right)\right] = \frac{\beta}{\eta} E_t\left[\frac{1}{c_{t+1}}\left(r_{t+1} + \frac{1-\delta}{V_{t+1}} + \eta^2 q\left(\frac{k_{t+1}}{k_t} - 1\right) + \eta^2 q\frac{\left(k_{t+1} - k_t\right)^2}{2k_t^2}\right)\right] \tag{38}$$

$$\frac{1}{c_t} = \omega p_t + \beta E_t\left(\frac{p_t}{c_{t+1} p_{t+1}}\right) \tag{39}$$

$$mc_t = r_t^{\theta} w_t^{1-\theta} A_t^{-1}\left(\frac{1}{1-\theta}\right)^{1-\theta}\theta^{-\theta} \tag{40}$$

$$\frac{w_t}{r_t} = \left(\frac{1-\theta}{\theta}\right)\frac{k_{t-1}}{h_t} \tag{41}$$

$$p_{j,t}^* = \frac{\xi E_t \sum_{k=0}^{\infty} (\alpha\beta)^k \frac{y_{t+k}}{c_{t+k}} p_{t+k}^{\xi} mc_{t+k}}{(\xi - 1) E_t \sum_{k=0}^{\infty} (\alpha\beta)^k \frac{y_{t+k}}{c_{t+k}} p_{t+k}^{\xi-1}} \tag{42}$$

$$p_t^{1-\xi} = (1-\alpha) p_{j,t}^{*1-\xi} + \alpha p_{t-1}^{1-\xi}\eta^{1-\xi} \tag{43}$$

$$A_t k_{t-1}^{\theta} H_t^{1-\theta} = g_t + c_t + i_t + q\frac{\left(k_t - k_{t-1}\right)^2}{2k_{t-1}^2} \tag{44}$$

$$i_t = \frac{\eta k_t - (1-\delta) k_{t-1}}{V_t} \tag{45}$$

$$\ln A_t = \rho_A \ln A_{t-1} + (1-\rho_A)\ln A + \varepsilon_t^A \tag{46}$$

$$\ln g_t = \rho_g \ln g_{t-1} + (1-\rho_g)\ln g + \varepsilon_t^g \tag{47}$$

$$\ln V_t = \rho_V \ln V_{t-1} + (1-\rho_V)\ln V + \varepsilon_t^V \tag{48}$$

2.3.8 Steady State

In the steady state, we have

$$r = \frac{\eta}{\beta} - 1 + \delta \tag{49}$$

$$\frac{k}{y} = \frac{\theta}{r} \tag{50}$$

$$\frac{i}{y} = (\eta - 1 + \delta)\frac{k}{y} \tag{51}$$

$$\frac{c}{y} = 1 - \frac{g}{y} - \frac{i}{y}. \tag{52}$$

2.3.9 Log-linearized Equilibrium

Redefining variables in terms of their percent deviations from steady state, the log-linearized equilibrium conditions are

$$w_t = c_t \tag{53}$$

$$-c_t + E_t c_{t+1} + \frac{\eta^2 q}{r+1-\delta} E_t k_{t+1} = \frac{r}{r+1-\delta} E_t r_{t+1} - \frac{1-\delta}{r+1-\delta}\rho_V V_t + q\eta k_{t-1} - q\eta\left(1 + \frac{\eta}{r+1-\delta}\right) k_t \tag{54}$$

$$\beta E_t c_{t+1} + \beta E_t p_{t+1} = p_t + c_t \tag{55}$$

$$mc_t = \theta r_t + (1-\theta) w_t - A_t \tag{56}$$

$$w_t - r_t = k_{t-1} - h_t \tag{57}$$

$$p_t = a_1 mc_t + a_2 p_{t-1} + a_3 E_t p_{t+1} \tag{58}$$

$$A_t + \theta k_{t-1} + (1-\theta) H_t = \frac{c}{y} c_t + \eta \frac{k}{y} k_t - (1-\delta)\frac{k}{y} k_{t-1} + (\eta - 1 + \delta)\frac{k}{y} V_t + \frac{g}{y} g_t \tag{59}$$

$$A_t = \rho_A A_{t-1} + \varepsilon_t^A \tag{60}$$

$$g_t = \rho_g g_{t-1} + \varepsilon_t^g \tag{61}$$

$$V_t = \rho_V V_{t-1} + \varepsilon_t^V \tag{62}$$

where

$$a_1 = \frac{(1-\alpha)(1-\beta)}{\alpha\left(1 + \beta - \alpha\beta + \alpha\beta\eta^{1-\xi}\right)}$$

$$a_2 = \frac{\beta}{1 + \beta - \alpha\beta + \alpha\beta\eta^{1-\xi}}$$

$$a_3 = \frac{\eta^{1-\xi}}{1 + \beta - \alpha\beta + \alpha\beta\eta^{1-\xi}}.$$

2.3.10 Putting the Model in Blanchard-Kahn Form

Substituting (56) into (58)

$$p_t = a_1 c_t - a_1 \theta k_{t-1} + a_1 \theta H_t - a_1 A_t + a_2 p_{t-1} + a_3 E_t p_{t+1}. \tag{63}$$

Combining (53) with (57) and substituting the resulting definition or r_{t+1} into (54), we have

$$E_t c_{t+1} + \frac{r}{r+1-\delta} k_t - \frac{r}{r+1-\delta} E_t H_{t+1} - \frac{\eta^2 q}{r+1-\delta} k_{t+1}$$
$$= \left(1 + \frac{r}{r+1-\delta}\right) c_t + \left(\frac{r}{r+1-\delta}\right)\rho_V V_t - \left(\frac{r + q\eta^2}{r+1-\delta} + q\eta\right) k_t + q\eta k_{t-1}. \tag{64}$$

Then substitute out for $E_t H_{t+1}$ in (63) and (64) using (59). Then define $D_t^1 = p_{t-1}$, $D_t^2 = k_{t-1}$, $s_t^0 = \left[D_t^1, D_t^2, k_t, p_t, c_t, H_t\right]'$, $z_t^0 = [A_t, g_t, V_t]$. Now we can write the model as

$$E_t s_{t+1}^0 = A^{-1} B s_t^0 + A^{-1} C$$

where

$$\begin{aligned} A &= \begin{bmatrix} 0 & 0 & 0 & \beta & \beta \\ 0 & 0 & 0 & -b_4 & 0 \\ 0 & 0 & b_{10} & 0 & b_9 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix}, \\ B &= \begin{bmatrix} 0 & 0 & 0 & 1 & 1 \\ b_3 & b_2 & b_5 & -1 & b_1 \\ 0 & b_{12} & b_{13} & 0 & b_{11} \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}, \\ C &= \begin{bmatrix} 0 & 0 & 0 \\ b_6 & b_7 & b_8 \\ b_{14} & b_{15} & b_{16} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \end{aligned}$$

and

$$
\begin{aligned}
b_1 &= a_1\left(1+\frac{\theta c/y}{1-\theta}\right)\\
b_2 &= -a_1\theta\left[1+\frac{\theta+(1-\delta)\,k/y}{1-\theta}\right]\\
b_3 &= a_2\\
b_4 &= a_3\\
b_5 &= a_1\theta\frac{\eta}{1-\theta}\frac{k}{y}\\
b_6 &= -a_1\left[1+\frac{\theta}{1-\theta}\right]\\
b_7 &= a_1\theta\frac{g/y}{1-\theta}\\
b_8 &= a_1\theta\frac{\eta-1+\delta}{1-\theta}\frac{k}{y}\\
b_9 &= 1-\frac{r}{1+r-\delta}\left(\frac{c/y}{1-\theta}\right)\\
b_{10} &= -\left[\frac{r}{1+r-\delta}\frac{\eta}{1-\theta}\frac{k}{y}+\frac{\eta^2 q}{1+r-\delta}\right]\\
b_{11} &= 1+\frac{r}{1+r-\delta}\\
b_{12} &= q\eta\\
b_{13} &= -\frac{1}{1+r-\delta}\left[r+q\eta^2+r\left(\frac{\theta+(1-\delta)\frac{k}{y}}{1-\theta}\right)\right]-q\eta\\
b_{14} &= -\frac{r}{1+r-\delta}\frac{\rho_A}{1-\theta}\\
b_{15} &= \frac{r}{1+r-\delta}\frac{g/y}{1-\theta}\rho_g\\
b_{16} &= 1-\frac{1-\delta}{1+r-\delta}\rho_V+\frac{r}{1+r-\delta}\left(\frac{\eta-1+\delta}{1-\theta}\right)\frac{k}{y}\rho_V.
\end{aligned}
$$

The remainder of the model solutions uses the same process as in the two models above.

2.4 A Sticky Price Model with an Accomodating Monetary Authority

The model is the same as that of section 1.3 except that there is now an additional equilibrium condition for the money supply,

$$M_{t+1}=\rho_M M_t+\rho_A A_t,$$

such that the log-linearized intertemporal Euler for real balances becomes

$$\beta E_t c_{t+1}+\beta E_t p_{t+1}+(1-\beta)\,M_t=c_t+p_t.$$

Define a_1, a_2, and a_3 as in section 1.3 and again substitute the marginal cost equation into the equation for the price level we get

$$p_t=a_1c_t-a_1\theta k_{t-1}+a_1\theta H_t-a_1A_t+a_2p_{t-1}+a_3E_tp_{t+1} \tag{65}$$

and substituting r_{t+1} from the cost minimzation equation into the intertemporal Euler for capital we get

$$\begin{aligned} E_t c_{t+1} - \frac{r}{r+1-\delta} E_t H_{t+1} - \frac{\eta^2 q}{r+1-\delta} E_t k_{t+1} &= \left[1 + \frac{r}{r+1-\delta}\right] c_t + \left[1 - \frac{1-\delta}{r+1-\delta}\rho_V\right] V_t \\ &\quad - \left[\frac{r+q\eta^2}{r+1-\delta} + q\eta\right] k_t + q\eta k_{t-1}. \end{aligned} \tag{66}$$

Then substitute for H_t from the resource constraint,

$$H_t = \frac{c/y}{1-\theta} c_t + \frac{\eta}{1-\theta}\frac{k}{y} k_t - \frac{\left[\theta + (1-\delta)\frac{k}{y}\right]}{1-\theta} k_{t-1} + \frac{\eta - 1 + \delta}{1-\theta}\frac{k}{y} V_t + \frac{g/y}{1-\theta} g_t - \frac{1}{1-\theta} A_t, \tag{67}$$

and write

$$\begin{aligned} p_t &= b_1 c_t + b_2 k_{t-1} + b_3 p_{t-1} + b_4 E_t p_{t+1} + b_5 k_t + b_6 A_t + b_7 g_t + b_8 V_t \qquad (68) \\ b_1 &= a_1 \left(1 + \frac{\theta c/y}{1-\theta}\right) \\ b_2 &= -a_1 \theta \left[1 + \frac{\theta + (1-\delta) k/y}{1-\theta}\right] \\ b_3 &= a_2 \\ b_4 &= a_3 \\ b_5 &= a_1 \theta \frac{\eta}{1-\theta}\frac{k}{y} \\ b_6 &= -a_1 \left(1 + \frac{\theta}{1-\theta}\right) \\ b_7 &= a_1 \theta \frac{g/y}{1-\theta} \\ b_8 &= a_1 \theta \frac{\eta - 1 + \delta}{1-\theta}\frac{k}{y} \end{aligned}$$

and

$$\begin{aligned} b_9 E_t c_{t+1} + b_{10} E_t k_{t+1} &= b_{11} c_t + b_{12} k_{t-1} + b_{13} k_t + b_{14} A_t + b_{15} g_t + b_{16} V_t \\ b_9 &= 1 - \frac{r}{1+r-\delta}\left(\frac{c/y}{1-\theta}\right) \\ b_{10} &= -\left[\frac{r}{1+r-\delta}\left(\frac{\eta k/y}{1-\theta}\right) + \frac{\eta^2 q}{r+1-\delta}\right] \\ b_{11} &= 1 + \frac{r}{1+r-\delta} \\ b_{12} &= q\eta \\ b_{13} &= -\frac{1}{1+r-\delta}\left[r + q\eta^2 + \frac{r(\theta + (1-\delta) k/y)}{1-\theta}\right] - q\eta \\ b_{14} &= -\frac{r}{1+r-\delta}\left(\frac{1}{1-\theta}\right)\rho_A \\ b_{15} &= \frac{r}{1+r-\delta}\left(\frac{g/y}{1-\theta}\right)\rho_g \\ b_{16} &= 1 - \frac{1-\delta}{1+r-\delta}\rho_V + \frac{r}{1+r-\delta}\left(\frac{\eta - 1 + \delta}{(1-\theta)}\right)\left(\frac{k}{y}\right)\rho_V. \end{aligned}$$

Now define $D_t^1 = p_{t-1}$, $D_t^2 = k_{t-1}$, $s_t^0 = \left[D_t^1, D_t^2, M_t, k_t, p_t, c_t\right]'$, $z_t^0 = [A_t, g_t, V_t]'$ and write the system as

$$AE_t s_{t+1}^0 = Bs_t^0 + Cz_t^0$$

where

$$\begin{aligned} A &= \begin{bmatrix} 0 & 0 & 0 & 0 & \beta & \beta \\ 0 & 0 & 0 & 0 & -b_4 & 0 \\ 0 & 0 & 0 & b_{10} & 0 & b_9 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}, \\ B &= \begin{bmatrix} 0 & 0 & \beta - 1 & 0 & 1 & 1 \\ b_3 & b_2 & 0 & b_5 & -1 & b_1 \\ 0 & b_{12} & 0 & b_{13} & 0 & b_{11} \\ 0 & 0 & \rho_M & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}, \\ C &= \begin{bmatrix} 0 & 0 & 0 \\ b_6 & b_7 & b_8 \\ b_{14} & b_{15} & b_{16} \\ \rho_A & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}. \end{aligned}$$

Then we have

$$E_t s_{t+1}^0 = Ds_t^0 + A^{-1}Cz_t^0$$

where

$$D = A^{-1}B.$$

Letting $D = G^{-1}HG$ and $J = GA^{-1}C$, we can write the system as

$$E_t s_{1,t+1}^1 = \lambda_{1H} s_{1,t}^1 + J_{11}A_t + J_{12}g_t + J_{13}V_t \tag{69}$$
$$E_t s_{2,t+1}^1 = \lambda_{2H} s_{2,t}^1 + J_{21}A_t + J_{22}g_t + J_{23}V_t \tag{70}$$
$$E_t s_{3,t+1}^1 = \lambda_{3H} s_{3,t}^1 + J_{31}A_t + J_{32}g_t + J_{33}V_t \tag{71}$$
$$E_t s_{4,t+1}^1 = \lambda_{4H} s_{4,t}^1 + J_{41}A_t + J_{42}g_t + J_{43}V_t \tag{72}$$
$$E_t s_{5,t+1}^1 = \lambda_{5H} s_{5,t}^1 + J_{51}A_t + J_{52}g_t + J_{53}V_t \tag{73}$$
$$E_t s_{6,t+1}^1 = \lambda_{6H} s_{1,t}^1 + J_{61}A_t + J_{62}g_t + J_{63}V_t \tag{74}$$

where

$$\begin{aligned} s_{1,t}^1 &= G_{11}D_t^1 + G_{12}D_t^2 + G_{13}M_t + G_{14}k_t + G_{15}p_t + G_{16}c_t \\ s_{2,t}^1 &= G_{21}D_t^1 + G_{22}D_t^2 + G_{23}M_t + G_{24}k_t + G_{25}p_t + G_{26}c_t \\ s_{3,t}^1 &= G_{31}D_t^1 + G_{32}D_t^2 + G_{33}M_t + G_{34}k_t + G_{35}p_t + G_{36}c_t \\ s_{4,t}^1 &= G_{41}D_t^1 + G_{42}D_t^2 + G_{43}M_t + G_{44}k_t + G_{45}p_t + G_{46}c_t \\ s_{5,t}^1 &= G_{51}D_t^1 + G_{52}D_t^2 + G_{53}M_t + G_{54}k_t + G_{55}p_t + G_{56}c_t \\ s_{6,t}^1 &= G_{61}D_t^1 + G_{62}D_t^2 + G_{63}M_t + G_{64}k_t + G_{65}p_t + G_{66}c_t. \end{aligned}$$

Since D_t^1, D_t^2, and M_t are predetermined at t, $|\lambda_{4H}|, |\lambda_{5H}|, |\lambda_{6H}| < 1$, we have

$$\begin{aligned}
G_{41}D_t^1 + G_{42}D_t^2 + G_{43}M_t + G_{44}k_t + G_{45}p_t + G_{46}c_t &= \frac{J_{41}}{\rho_A - \lambda_{4H}}A_t + \frac{J_{42}}{\rho_g - \lambda_{4H}}g_t + \frac{J_{43}}{\rho_V - \lambda_{4H}}V_t \\
G_{51}D_t^1 + G_{52}D_t^2 + G_{53}M_t + G_{54}k_t + G_{55}p_t + G_{56}c_t &= \frac{J_{51}}{\rho_A - \lambda_{5H}}A_t + \frac{J_{52}}{\rho_g - \lambda_{5H}}g_t + \frac{J_{53}}{\rho_V - \lambda_{5H}}V_t \\
G_{61}D_t^1 + G_{62}D_t^2 + G_{63}M_t + G_{64}k_t + G_{65}p_t + G_{66}c_t &= \frac{J_{61}}{\rho_A - \lambda_{6H}}A_t + \frac{J_{62}}{\rho_g - \lambda_{6H}}g_t + \frac{J_{63}}{\rho_V - \lambda_{6H}}V_t
\end{aligned}$$

which we can write as

$$\begin{bmatrix} k_t \\ p_t \\ c_t \end{bmatrix} = N \begin{bmatrix} D_t^1 \\ D_t^2 \\ M_t \end{bmatrix} + O z_t^0$$

where

$$\begin{aligned}
N &= K^{-1}L,\ O = K^{-1}M \\
K &= \begin{bmatrix} G_{44} & G_{45} & G_{46} \\ G_{54} & G_{55} & G_{56} \\ G_{64} & G_{65} & G_{66} \end{bmatrix} \\
L &= -\begin{bmatrix} G_{41} & G_{42} & G_{43} \\ G_{51} & G_{52} & G_{53} \\ G_{61} & G_{62} & G_{63} \end{bmatrix} \\
M &= \begin{bmatrix} \frac{J_{41}}{\rho_A - \lambda_{4H}} & \frac{J_{42}}{\rho_g - \lambda_{4H}} & \frac{J_{43}}{\rho_V - \lambda_{4H}} \\ \frac{J_{51}}{\rho_A - \lambda_{5H}} & \frac{J_{52}}{\rho_g - \lambda_{5H}} & \frac{J_{53}}{\rho_V - \lambda_{5H}} \\ \frac{J_{61}}{\rho_A - \lambda_{6H}} & \frac{J_{62}}{\rho_g - \lambda_{6H}} & \frac{J_{63}}{\rho_V - \lambda_{6H}} \end{bmatrix}.
\end{aligned}$$

We can now write (69), (70), and (71) as

$$\begin{aligned}
& G_{11}E_tD_{t+1}^1 + G_{12}E_tD_{t+1}^2 + G_{13}E_tM_{t+1} + \\
& G_{14}N_{11}E_tD_{t+1}^1 + G_{14}N_{12}E_tD_{t+1}^2 + G_{14}N_{13}E_tM_{t+1} + G_{14}O_{11}\rho_A A_t + G_{14}O_{12}\rho_g g_t + G_{14}O_{13}\rho_V V_t + \\
& G_{15}N_{21}E_tD_{t+1}^1 + G_{15}N_{22}E_tD_{t+1}^2 + G_{15}N_{23}E_tM_{t+1} + G_{15}O_{21}\rho_A A_t + G_{15}O_{22}\rho_g g_t + G_{15}O_{23}\rho_V V_t + \\
& G_{16}N_{31}E_tD_{t+1}^1 + G_{16}N_{32}E_tD_{t+1}^2 + G_{16}N_{33}E_tM_{t+1} + G_{16}O_{31}\rho_A A_t + G_{16}O_{32}\rho_g g_t + G_{16}O_{33}\rho_V V_t \\
= \; & \lambda_{1H}\left(G_{11} + G_{14}N_{11} + G_{15}N_{21} + G_{16}N_{31}\right)D_t^1 + \lambda_{1H}\left(G_{12} + G_{14}N_{12} + G_{15}N_{22} + G_{16}N_{32}\right)D_t^2 \\
& + \lambda_{1H}\left(G_{13} + G_{14}N_{13} + G_{15}N_{23} + G_{16}N_{33}\right)M_t + \left[\lambda_{1H}\left(G_{14}O_{11} + G_{15}O_{21} + G_{16}O_{31}\right) + J_{11}\right]A_t \\
& + \left[\lambda_{1H}\left(G_{14}O_{12} + G_{15}O_{22} + G_{16}O_{32}\right) + J_{12}\right]g_t + \left[\lambda_{1H}\left(G_{14}O_{13} + G_{15}O_{23} + G_{16}O_{33}\right) + J_{13}\right]V_t,
\end{aligned}$$

$$\begin{aligned}
& G_{21}E_tD_{t+1}^1 + G_{22}E_tD_{t+1}^2 + G_{23}E_tM_{t+1} + \\
& G_{24}N_{11}E_tD_{t+1}^1 + G_{24}N_{12}E_tD_{t+1}^2 + G_{24}N_{13}E_tM_{t+1} + G_{24}O_{11}\rho_A A_t + G_{24}O_{12}\rho_g g_t + G_{24}O_{13}\rho_V V_t + \\
& G_{25}N_{21}E_tD_{t+1}^1 + G_{25}N_{22}E_tD_{t+1}^2 + G_{25}N_{23}E_tM_{t+1} + G_{25}O_{21}\rho_A A_t + G_{25}O_{22}\rho_g g_t + G_{25}O_{23}\rho_V V_t + \\
& G_{26}N_{31}E_tD_{t+1}^1 + G_{26}N_{32}E_tD_{t+1}^2 + G_{26}N_{33}E_tM_{t+1} + G_{26}O_{31}\rho_A A_t + G_{26}O_{32}\rho_g g_t + G_{26}O_{33}\rho_V V_t \\
= \; & \lambda_{2H}\left(G_{21} + G_{24}N_{11} + G_{25}N_{21} + G_{26}N_{31}\right)D_t^1 + \lambda_{2H}\left(G_{22} + G_{24}N_{12} + G_{25}N_{22} + G_{26}N_{32}\right)D_t^2 \\
& + \lambda_{2H}\left(G_{23} + G_{24}N_{13} + G_{25}N_{23} + G_{26}N_{33}\right)M_t + \left[\lambda_{2H}\left(G_{24}O_{11} + G_{25}O_{21} + G_{26}O_{31}\right) + J_{21}\right]A_t \\
& + \left[\lambda_{2H}\left(G_{24}O_{12} + G_{25}O_{22} + G_{26}O_{32}\right) + J_{22}\right]g_t + \left[\lambda_{2H}\left(G_{24}O_{13} + G_{25}O_{23} + G_{26}O_{33}\right) + J_{23}\right]V_t,
\end{aligned}$$

and

$$
\begin{aligned}
& G_{31}E_tD^1_{t+1} + G_{32}E_tD^2_{t+1} + G_{33}E_tM_{t+1} + \\
& G_{34}N_{11}E_tD^1_{t+1} + G_{34}N_{12}E_tD^2_{t+1} + G_{34}N_{13}E_tM_{t+1} + G_{34}O_{11}\rho_A A_t + G_{34}O_{12}\rho_g g_t + G_{34}O_{13}\rho_V V_t + \\
& G_{35}N_{21}E_tD^1_{t+1} + G_{35}N_{22}E_tD^2_{t+1} + G_{35}N_{23}E_tM_{t+1} + G_{35}O_{21}\rho_A A_t + G_{35}O_{22}\rho_g g_t + G_{35}O_{23}\rho_V V_t + \\
& G_{36}N_{31}E_tD^1_{t+1} + G_{36}N_{32}E_tD^2_{t+1} + G_{36}N_{33}E_tM_{t+1} + G_{36}O_{31}\rho_A A_t + G_{36}O_{32}\rho_g g_t + G_{36}O_{33}\rho_V V_t \\
= \; & \lambda_{3H}\left(G_{31} + G_{34}N_{11} + G_{35}N_{21} + G_{36}N_{31}\right)D^1_t + \lambda_{3H}\left(G_{32} + G_{34}N_{12} + G_{35}N_{22} + G_{36}N_{32}\right)D^2_t \\
& + \lambda_{3H}\left(G_{33} + G_{34}N_{13} + G_{35}N_{23} + G_{36}N_{33}\right)M_t + \left[\lambda_{3H}\left(G_{34}O_{11} + G_{35}O_{21} + G_{36}O_{31}\right) + J_{31}\right]A_t \\
& + \left[\lambda_{3H}\left(G_{34}O_{12} + G_{35}O_{22} + G_{36}O_{32}\right) + J_{32}\right]g_t + \left[\lambda_{3H}\left(G_{34}O_{13} + G_{35}O_{23} + G_{36}O_{33}\right) + J_{33}\right]V_t.
\end{aligned}
$$

Letting $s_t = \left[D^1_t, D^2_t, M_t, A_t, g_t, V_t\right]'$, and $\varepsilon_t = \left[\varepsilon^A_t, \varepsilon^g_t, \varepsilon^V_t\right]'$ we can write the system as

$$s_{t+1} = \Pi s_t + W\varepsilon_t$$

where

$$\Pi = P^{-1}Q,$$

$$
\begin{aligned}
P_{11} &= G_{11} + G_{14}N_{11} + G_{15}N_{21} + G_{16}N_{31} \\
P_{12} &= G_{12} + G_{14}N_{12} + G_{15}N_{22} + G_{16}N_{32} \\
P_{13} &= G_{13} + G_{14}N_{13} + G_{15}N_{23} + G_{16}N_{33} \\
P_{21} &= G_{21} + G_{24}N_{11} + G_{25}N_{21} + G_{26}N_{31} \\
P_{22} &= G_{22} + G_{24}N_{12} + G_{25}N_{22} + G_{26}N_{32} \\
P_{23} &= G_{23} + G_{24}N_{13} + G_{25}N_{23} + G_{26}N_{33} \\
P_{31} &= G_{31} + G_{34}N_{11} + G_{35}N_{21} + G_{36}N_{31} \\
P_{32} &= G_{32} + G_{34}N_{12} + G_{35}N_{22} + G_{36}N_{32} \\
P_{33} &= G_{33} + G_{34}N_{13} + G_{35}N_{23} + G_{36}N_{33} \\
P_{44} &= 1 \\
P_{55} &= 1 \\
P_{66} &= 1,
\end{aligned}
$$

$$
\begin{aligned}
Q_{11} &= \lambda_{1H} P_{11} \\
Q_{12} &= \lambda_{1H} P_{12} \\
Q_{13} &= \lambda_{1H} P_{13} \\
Q_{14} &= J_{11} + (\lambda_{1H} - \rho_A)(G_{14} O_{11} + G_{15} O_{21} + G_{16} O_{31}) \\
Q_{15} &= J_{12} + (\lambda_{1H} - \rho_g)(G_{14} O_{12} + G_{15} O_{22} + G_{16} O_{32}) \\
Q_{16} &= J_{13} + (\lambda_{1H} - \rho_V)(G_{14} O_{13} + G_{15} O_{23} + G_{16} O_{33}) \\
Q_{21} &= \lambda_{2H} P_{21} \\
Q_{22} &= \lambda_{2H} P_{22} \\
Q_{23} &= \lambda_{2H} P_{23} \\
Q_{24} &= J_{21} + (\lambda_{2H} - \rho_A)(G_{24} O_{11} + G_{25} O_{21} + G_{26} O_{31}) \\
Q_{25} &= J_{22} + (\lambda_{2H} - \rho_g)(G_{24} O_{12} + G_{25} O_{22} + G_{26} O_{32}) \\
Q_{26} &= J_{23} + (\lambda_{2H} - \rho_V)(G_{24} O_{13} + G_{25} O_{23} + G_{26} O_{33}) \\
Q_{31} &= \lambda_{3H} P_{31} \\
Q_{32} &= \lambda_{3H} P_{32} \\
Q_{33} &= \lambda_{3H} P_{33} \\
Q_{34} &= J_{31} + (\lambda_{3H} - \rho_A)(G_{34} O_{11} + G_{35} O_{21} + G_{36} O_{31}) \\
Q_{35} &= J_{32} + (\lambda_{3H} - \rho_g)(G_{34} O_{12} + G_{35} O_{22} + G_{36} O_{32}) \\
Q_{36} &= J_{33} + (\lambda_{3H} - \rho_V)(G_{34} O_{13} + G_{35} O_{23} + G_{36} O_{33}) \\
Q_{44} &= \rho_A \\
Q_{55} &= \rho_g \\
Q_{66} &= \rho_V
\end{aligned}
$$

and

$$
W = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.
$$

Finally, let $f_t = [y_t, i_t, H_t, c_t]'$, $v_t = [\varepsilon_t^y, 0, 0, 0]'$ and write

$$
f_t = U s_t + v_t
$$

$$
U = \begin{bmatrix} (1-\theta)\,t_1 & \theta + (1-\theta)\,t_2 & (1-\theta)\,t_3 & (1-\theta)\,t_4 + 1 & (1-\theta)\,t_5 & (1-\theta)\,t_6 \\ t_7 & t_8 & t_9 & t_{10} & t_{11} & t_{12} \\ t_1 & t_2 & t_3 & t_4 & t_5 & t_6 \\ N_{31} & N_{32} & N_{33} & O_{31} & O_{32} & O_{33} \end{bmatrix}
$$

where

$$
\begin{aligned}
t_1 &= \frac{c/y}{1-\theta} N_{31} + \frac{\eta k/y}{1-\theta} \Pi_{21} \\
t_2 &= \frac{c/y}{1-\theta} N_{32} + \frac{\eta k/y}{1-\theta} \Pi_{22} - \frac{[\theta + (1-\delta)\, k/y]}{1-\theta} \\
t_3 &= \frac{c/y}{1-\theta} N_{33} + \frac{\eta k/y}{1-\theta} \Pi_{23} \\
t_4 &= \frac{c/y}{1-\theta} O_{31} + \frac{\eta k/y}{1-\theta} \Pi_{24} - \frac{1}{1-\theta} \\
t_5 &= \frac{c/y}{1-\theta} O_{32} + \frac{\eta k/y}{1-\theta} \Pi_{25} + \frac{g/y}{1-\theta} \\
t_6 &= \frac{c/y}{1-\theta} O_{33} + \frac{\eta k/y}{1-\theta} \Pi_{26} + \frac{\eta - 1 + \delta}{1-\theta} \left(\frac{k}{y} \right) \\
t_7 &= \frac{\eta}{\eta - 1 + \delta} \Pi_{21} \\
t_8 &= \frac{\eta \Pi_{22} - 1 + \delta}{\eta - 1 + \delta} \\
t_9 &= \frac{\eta}{\eta - 1 + \delta} \Pi_{23} \\
t_{10} &= \frac{\eta}{\eta - 1 + \delta} \Pi_{24} \\
t_{11} &= \frac{\eta}{\eta - 1 + \delta} \Pi_{25} \\
t_{12} &= \frac{\eta}{\eta - 1 + \delta} \Pi_{26} - 1.
\end{aligned}
$$