Second Mortgages: Valuation and Implications for the Performance of Structured Financial Products*

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Abstract

We provide an analytic valuation framework to value second lien mortgages and first lien mortgages when homeowners can take out a second lien. We then use the framework to value mortgage backed securities (MBS) and, in particular, quantify the greater risk associated with MBS backed by first liens that have "silent seconds". Rating MBS without accounting for homeowners' equity extraction option results in much higher ratings than warranted by expected loss. While in our benchmark calibration the senior tranche rating should be $A1$ rather than Aaa, the big losers from the equity extraction option are the mezzanine tranches which are nearly wiped out.

Keywords: Mortgage Backed Securities (MBS); Mortgage Valuation; Credit Ratings.

JEL Classification: G12, G21, G23, G24.

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1 Introduction

In contrast to most types of debt, including corporate bonds and commercial mortgages, residential mortgagors retain the option to take on subsequent debt. Indeed, the sharp rise in US home prices between 2001 and 2006 coincided with a substantially greater use of second liens and was a key way home owners increased their leverage throughout the housing boom.[1] Goodman et al. (2010) calculate that more than 50% of first liens in private label securitizations over the 2000 to 2007 time period had a second lien behind them, obtained either subsequently as second mortgages or simultaneously in the form of piggy-back financing. Sherlund (2008), Piskorski et al. (2015), and Griffin and Maturana (2016) document that piggy-back financing in particular is associated with more defaults of first lien mortgages. Given the prominence of second liens in the subprime financial crisis, and the risk that second liens also expose the Government Sponsored Entitities (GSEs) to, it is important to understand their implications for the valuation of first lien mortgages and, in turn, the properties of structured financial products, like mortgage backed securities (MBS), collateralized by these mortgages.

To do so, we provide, in the spirit of Black and Cox (1976), a closed-form structural model to value first liens as well as subordinated mortgages when homeowners can take on additional debt by extracting equity from properties that have appreciated. Unlike previous risky mortgage valuation models, we do not exogenously specify property values and their dynamics. Rather, we take a property's service flow, that is the rent on the property, as our state variable and endogenously derive property values as well as both senior and junior mortgage values. The role of a property's service flow in our model is analogous to that of a firm's *EBIT* in dynamic capital structure models (see, for example, Goldstein et al. (2001)). To our knowledge, ours is the first analytic model of junior mortgage liens.

[1] Keys et al. (2013) show that, while loan-to-values (LTVs) on privately securitized first liens stabilized by around 2003, the combined LTV (CLTV) on such loans rose by 10 percentage points between 2001 and 2006. More generally, Greenspan and Kennedy (2008) show that, during approximately the same time period, U.S. homeowners extracted an average of slightly under $700 billion of equity each year relying on cash-out refinancing, home equity lines-of-credit, and second mortgages.

We then use our structural valuation framework to investigate how the option to take on a second lien affects MBS collateralized by first lien mortgages. We do so because the bursting of the U.S. housing bubble saw the unraveling of many private label MBS. Some observers have argued that these large downgrades reflected the fact that credit rating agencies (CRAs) were blind to the possibility that first lien borrowers could subsequently obtain second liens, so-called "silent seconds" and, as a result, did not recognize the consequences of equity extraction on the performance of MBS.

To investigate this possibility, we posit a naïve CRA that rates an MBS ignoring the possibility that first lien mortgagors can obtain second liens. We consider a cash MBS collateralized by a pool of first lien mortgages. When we confront the resultant MBS structure with data generated by homeowners who optimally extract equity as well as default, we find that the resultant MBS performance is broadly consistent with the magnitude of downgrades observed subsequent to the bursting of the U.S. housing bubble. Furthermore, we find that it is the junior tranches of the MBS that are most significantly affected. In our benchmark calibration, simulations show that the true expected loss of a Aaa security sized based on a model without equity extraction is four notches (or two full grades) lower at $A1$. However, the mezzanine tranche, which we size to correspond to a $Baa3$ rating (corresponding to $BBB-$ on the S&P rating scale), is nearly completely wiped out when equity extraction is permitted.

In addition, our results do not support the argument that the downgrades observed in practice occurred only because the severity of the U.S. housing market downturn was simply underestimated. The distortion in ratings caused by equity extraction is more severe than the difference between *ex ante* ratings and *ex post* losses due to a realized bad aggregate home price scenario. In fact, we find that, absent equity extraction, the senior tranches would have preserved their Aaa ratings even for the aggregate U.S. home price paths realized following the worst MBS origination years.

The plan of this paper is as follows. The next section puts forward and details the properties of a closed-form structural model to value risky residential mortgages. We begin by allowing homeowners to only optimally default. In particular, homeowners pursue a *static* financing policy in which they rely on an exogenously specified loan-to-value ratio when originally purchasing their property. With subsequent property price appreciation, however, homeowners cannot extract equity by obtaining a second mortgage. Next we allow homeowners to extract equity in addition to optimally defaulting. Under a *dynamic* financing policy, homeowners now get a second lien when home prices appreciate sufficiently. Section 3 investigates the extent to which the unraveling of MBS in the aftermath of the bursting of the U.S. housing bubble can be attributed to naïve CRAs who ignored the presence of second liens. Section 4 concludes the paper.

2 Closed-Form Valuation of Risky Mortgages

Our underlying state variable is the service flow from a unit of property, denoted by δ, which represents the cost per unit of time of renting the property. The role of δ in our model is analogous to that of a firm's *EBIT* in dynamic capital structure models (see, for example, Goldstein et al. (2001)). This is in contrast to the traditional approach of valuing risky mortgages which takes an unlevered property value as a state variable.[2] Our approach views real estate itself as a contingent claim on δ which can then be valued alongside the risky mortgage. The effects of changing mortgage features on property values can be easily explored within this framework.[3]

The dynamics of δ are given by

$$d\delta_t = \delta_t \mu dt + \delta_t \sigma dW_t \tag{1}$$

and, without loss of generality, we fix $\delta_0 = 1$. Here μ denotes the (instantaneous) drift of

[2]See, for example, Titman and Torous (1989), Kau et al. (1995), and Deng et al. (2000).

[3]For example, changes in maximum permitted loan-to-value ratios, higher foreclosure costs, the ability of property owners to take out a second lien, the imposition of transaction costs to dissuade second liens, etc.

the property service flow process while σ is its (instantaneous) volatility.

We make a number of simplifying assumptions in valuing claims contingent on δ. First, the homeowner finances an exogenously determined fraction ℓ of the property's purchase price by obtaining an infinite maturity mortgage requiring a fixed coupon payment rate of c. The reliance on mortgage financing reflects, for example, a tax advantage to debt or financing constraints which are not explicitly modeled. We thus exclude interest rate driven repayments. Second, the drift of the service flow process, μ is less than the risk free rate r. Otherwise the value of an infinite stream of service flow will be infinitely large.

The assumption of infinite maturity simplifies our analysis. Most actual mortgages have very long maturities, typically thirty years in the United States, and are often refinanced in order to extend their maturity. Because we apply our model to analyze the consequences on an MBS with a much shorter horizon, the assumption of an infinite maturity underlying mortgage is realistic and does not alter the main conclusions of our analysis.[4]

To keep the model simple we assume the prevailing risk free interest rate, r, is constant. Extending our model to stochastic interest rates would not alter our conclusions but would complicate the model substantially and make it more difficult to understand the intuition for our results. The role of interest rate changes in equity extraction is a separate question analyzed empirically by Bhutta and Keys (2016).

Finally, we abstract from the role of liquidity contraints and focus on what is known as strategic default in the credit risk literature. This implies that if the homeowner still has strictly positive value in her house, she will be able to find the liquidity to continue servicing the mortgage. Furthermore, the homeowner always defaults when the service flow hits the corresponding default boundary. In reality, homeowners are impacted by adverse liquidity shocks (for example, unemployment, divorce, or the death of a family member) that raise the marginal utility of consumption in the present and therefore increase the likelihood of

[4]However, we could model finite maturity in the form of an exponentially sinking fund feature by simply adding the (constant) sinking fund pay back rate to the interest rate as it is done in, for example, Leland (1998).

default. We abstract from the role of liquidity shocks to focus on the implications of home price changes.[5]

2.1 Debt and Equity Without Default or Second Liens

We denote the value of a mortgage by $D(\delta_t)$. The homeowner's residual claim on the property will be referred to as *equity* and denoted by $E(\delta_t)$. Assuming the homeowner never defaults then

$$\begin{aligned} E(\delta_t) &= \mathbb{E}_t\left[\int_t^\infty e^{-r(s-t)}(\delta_s - c)ds\right] \\ &= \int_t^\infty \left(e^{-r(s-t)}(\mathbb{E}_t[e^{ln\delta_s}] - c)\right)ds = \frac{\delta_t}{r-\mu} - \frac{c}{r} \qquad (2a) \\ D(\delta_t) &= \mathbb{E}_t\left[\int_t^\infty e^{-r(s-t)}cds\right] = \frac{c}{r}. \qquad (2b) \end{aligned}$$

In this case, the value of the house is simply the sum of the values of the mortgage and equity

$$D(\delta_t) + E(\delta_t) = \frac{\delta_t}{r-\mu}$$

and corresponds to the value obtained from a simplified version of a user cost of housing model.[6]

2.2 Permitting Default Only

Suppose now that the fixed rate mortgage is contractually defaultable and the homeowner cannot take out a second lien. We will refer to this as a static financing policy. It will serve as

[5]Our model could be extended to allow a role for liquidity shocks by adding an intensity based liquidity shock to the model. That is, when this liquidity shock hits (with intensity λ), the homeowner would default instantaneously. As long as this liquidity shock is modeled as a jump process with an intensity that is constant (or even as a function of the service flow) then this liquidity-based default can be modeled within our framework simply by adding the intensity, λ to the interest rate as in many reduced form credit risk models (see, for example, Duffie and Singleton (1999)).

[6]See, for example, Poterba (1984). The simplification stems from excluding depreciation, taxes, and maintenance costs. The user cost is the cost, including the opportunity cost, that an owner must pay to obtain a unit of housing services.

a benchmark against the later case of a dynamic financing policy in which homeowners can subsequently adjust the amount of debt outstanding to extract equity from their appreciated properties.

Since the mortgage has infinite maturity, we can find $E(\delta)$ and $D(\delta)$ by solving the corresponding standard risk-neutral pricing ordinary differential equations (see, for example, Goldstein et al. (2001)). For example, given the dynamics assumed for δ and using Itô's lemma, the capital gains to equity are given by

$$dE(\delta_t) = \mu\delta_t E'(\delta_t)dt + \delta_t\sigma E'(\delta_t)dW_t + \frac{1}{2}\sigma^2\delta_t^2 E''(\delta_t)dt$$

while the (instantaneous) dividend rate per unit of time is

$$\delta_t - c.$$

Under risk-neutral pricing, the standard ordinary differential equation (ODE) for equity is thus

$$\frac{1}{2}\sigma^2\delta_t^2 E''(\delta_t) + \mu\delta_t E'(\delta_t) - rE(\delta_t) + \delta_t - c = 0. \tag{3}$$

The general solutions for equity and debt are given by

$$E(\delta) = e\delta^{x_2} + \frac{\delta}{r-\mu} - \frac{c}{r} \tag{4a}$$

$$D(\delta) = d\delta^{x_2} + \frac{c}{r}, \tag{4b}$$

where

$$x_2 = \frac{(\frac{1}{2}\sigma^2 - \mu) - \sqrt{(\mu - \frac{1}{2}\sigma^2)^2 + 2r\sigma^2}}{\sigma^2} < 0$$

is the negative root of expression (3)'s associated quadratic equation while e and d are constants to be determined by initial and boundary conditions which characterize this valuation problem. We can exclude the term with a positive power greater than one in the general solutions, expressions (4a) and (4b), because we know that as δ approaches infinity these expressions must converge to the corresponding values calculated when default is not permitted, expressions (2a) and (2b).

The initial conditions describing the mortgage and equity at origination when $\delta_0 = \$1$ are given by

$$D(1) = P \tag{5a}$$

$$E(1) = A - P. \tag{5b}$$

Here P is the mortgage's principal and A is the value at origination of the underlying property financed by the mortgage.

The boundary conditions at the default boundary, $\delta = \delta_B$, are given by

$$E(\delta_B) = 0 \tag{6a}$$

$$E'(\delta_B) = 0 \tag{6b}$$

$$D(\delta_B) = (1 - \alpha)\delta_B A. \tag{6c}$$

The first boundary condition states that at default the homeowner's equity stake in the property is worthless. The corresponding smooth-pasting condition is given by the second boundary condition. The final boundary condition captures the fact that at default the lender receives the then prevailing value of the property $\delta_B A$, that is, the property value at origination scaled by the service flow level at default, all net of foreclosure costs where α is the exogenously specified percentage foreclosure loss.[7]

Because the property is infinitely lived, our valuation framework must make assumptions about its disposition subsequent to a default. We assume that foreclosure is immediate and the lender then sells the property for its prevailing value net of foreclosure costs to a buyer

[7] Implicit here and throughout this paper is the assumption that mortgage loans are non-recourse thereby limiting a lender's recovery to the property itself.

who again finances at a loan-to-value ratio of ℓ using a fixed rate infinite maturity mortgage.

Solving the risk-neutral pricing ordinary differential equation subject to these initial and boundary conditions determines the constants e and d as well the default boundary δ_B, the mortgage principal P and the house value at origination A. Finally, the mortgage's fixed coupon payment rate c is implicitly determined by solving

$$\frac{P}{A} = \ell. \tag{7}$$

2.2.1 Solution without Second Liens

We investigate the properties of the model for a base case specification of underlying parameter values. We then sequentially perturb a particular parameter value, holding all other parameter values unchanged, to gauge the model's resultant sensitivities. The results are tabulated in Table 1.

The base case sets the instantaneous drift of the property's service flow process at $\mu = 2\%$ and the instantaneous volatility, σ, at 5%. We fix the risk free rate at $r = 5\%$. We set the LTV at origination at $\ell = 80\%$ and assume that a foreclosure cost of $\alpha = 25\%$ of the then prevailing property value is incurred in the event of default. A value of α of this magnitude is in line with the empirical estimates of the foreclosure discount (see Campbell et al. (2011) and the review of earlier literature in Frame (2010)). In our case, α also captures administrative and legal costs associated with foreclosure.

We solve for the initial value of the home, A, the infinite maturity mortgage's principal, P, as well as its corresponding fixed coupon payment rate c. This results in an implied mortgage rate $y = c/P$. We also compute the level of the service flow which triggers default by the homeowner, δ_B. To gain additional insight into the likelihood of default or, alternatively, the expected length of time until default occurs, we also present the resultant equivalent fixed waiting time to default, EFWT, as well as the value of an Arrow-Debreu security contingent on default, ADD, which pays off \$1 only at default.[8]

From Table 1 we see that for the base case parameterization, the initial value of the home is A = \$33.28 while the homeowner borrows P = \$26.62 at a mortgage rate of $y = 5.01\%$ to obtain an 80% LTV. The homeowner subsequently finds it optimal to default when the property's service flow falls from δ_0 = \$1 to δ_B = \$0.76 which gives an equivalent fixed waiting time to default of EFWT = 96 years and an Arrow-Debreu security value contingent on default of ADD = \$0.008. Given our parameterization, default is a rare event for an LTV of 80% and the resultant default risk raises the cost of borrowing and lowers the property value only slightly.

The initial value of the home A is extremely sensitive to the prevailing risk free rate, r, largely reflecting the fact that it is the discounted value of an infinite stream of service flows. The resultant amount borrowed to maintain the $\ell = 80\%$ LTV varies correspondingly

[8]The expected waiting time until default is infinite for a geometric Brownian motion with positive drift ($\mu > 0$). In order to calculate a quantifiable measure of the waiting time until default, we use the value of an Arrow-Debreu security contingent on default defined by

$$\mathrm{ADD}(\delta_t) = \mathbb{E}_t\big[e^{-r(\tau_B - t)}\big]$$

where τ_B is the (stochastic) default time. We then define the *equivalent fixed waiting time to default* as the fixed waiting time into the future such that the value of receiving \$1 with certainty after this waiting time would be the same as the value of the Arrow-Debreu security contingent on default. That is, the equivalent fixed waiting time to default, $\mathrm{EFWT}(\delta)$, satisfies

$$\mathrm{ADD}(\delta) = e^{-r\,\mathrm{EFWT}(\delta)}$$

or

$$\mathrm{EFWT}(\delta) = -\ln(\mathrm{ADD}(\delta))/r.$$

The general solution for $ADD(\delta)$ is

$$\mathrm{ADD}(\delta) = add_1\delta^{x_1} + add_2\delta^{x_2}$$

where x_1 is the positive root. The two value matching conditions are

$$\lim_{\delta \uparrow \infty} ADD(\delta) = 0$$

and

$$ADD_0(\delta_B) = 1.$$

which gives the simple closed form solution

$$ADD(\delta) = \left(\frac{\delta}{\delta_B}\right)^{x_2}.$$

as does the mortgage rate. All else equal, default occurs sooner at a higher risk free rate (δ_B = \$0.757 for $r = 7\%$) as opposed to a lower risk free rate (δ_B = \$0.755 for $r = 3\%$). This reflects the basic property that American options are exercised sooner when interest rates are higher because the present value of waiting to exercise the option in the future is lower.

As the volatility of the property service flow process increases, the mortgage rate increases. For example, the mortgage rate increases from 5.00% at $\sigma = 3\%$, indicating a nearly riskless mortgage, to 5.09% at $\sigma = 7\%$. Default occurs sooner at a higher volatility but is triggered at a lower value of δ_B reflecting the greater likelihood of a rebound in the home's service flow when volatility is higher. Since foreclosure costs are capitalized in home values, higher foreclosure costs, α, result in a slightly lower initial property value A. A higher LTV, ℓ, means that default will occur sooner, also giving rise to a lower initial property value A and a substantially higher mortgage rate. This feature of the model is in contrast with models emphasizing the role of credit constraints in household behavior as well as empirical evidence.[9] Our model instead predicts lower property values because of the absence of credit constraints. We do not include credit constraints since our interest is in a tractable model of second liens rather than modeling home values. Consequently, our model highlights the deadweight costs of default that are reflected in home values.

2.3 Permitting Default and Second Liens

We now permit homeowners to take out a second lien as well as to default.[10] Homeowners follow a dynamic financing policy allowing them the option to extract equity by increasing their mortgage indebtedness in the event that property values rise. We are agnostic as to why a borrower wants to extract equity but the reason may be to pay off higher rate credit card

[9] See, for example Ortalo-Magné and Rady (2006) for a model of how LTVs affect property values in the presence of credit constraints. Favilukis et al. (forthcoming) presents a quantitative general equilibrium model with credit constraints showing that higher LTVs produce substantially higher equilibrium home values. Fuster and Zafar (2016) present survey evidence on the effects of LTV restrictions on home prices and review earlier empirical evidence.

[10] See the Appendix for the general case of n junior liens.

debt or to increase consumption.[11] A new buyer initially obtains a first lien mortgage with an LTV of ℓ_1. The subscript on ℓ indicates the number of extraction options available to the homeowner: 1 means one option is left while 0 indicates that no option to extract remains. To extract equity, the homeowner obtains a second mortgage in an amount incremental to the previous financing so as to give a *combined* LTV of ℓ_0 given the new higher property value. Like a closed-end second lien in the U.S., this incremental financing is assumed to be junior to all previous financing.[12] We consider the case where $\ell_0 = \ell_1$, such that the homeowner extracts equity only because property values have risen as well as the case where $\ell_0 > \ell_1$ which more closely resembles the case of "silent seconds".

Homeowners, however, do not decrease their mortgage indebtedness if property values fall. In our framework, the equity holder has no incentive to reduce her debt because of the classic debt overhang problem. As pointed out by Khandani et al. (2013), this "ratchet" effect also reflects the indivisible nature of real estate which implies that a homeowner cannot simply reduce leverage by selling a portion of the property and using the proceeds to reduce mortgage indebtedness. Furthermore, mortgage modification is difficult to accomplish in practice.[13]

We assume that a homeowner can take out a second lien at most one time over the course of owning a property.[14] The homeowner must determine the service flow, denoted by δ_F, at which to optimally take out the second lien. Analogous to the optimal exercise of an American option, the homeowner trades off locking in a certain gain from taking a second lien today versus waiting for an even larger gain at some future date. Lenders are aware of the homeowner's optimal strategy, and price mortgages accordingly.

[11]Labison et al. (2015) document that more than three quarters of US households pay *interest* on their credit cards every month at an average interest rate of 12%. Abdallah and Lastrapes (2012) show that a relaxation of legal restrications on home equity lending lead to a measurable increase in consumption.

[12]We model second liens as akin to home equity loans for residential properties for tractability but home equity lines of credit (HELOC) play a similar role in our framework. See Agarwal et al. (2006) for an empirical analysis of the difference between the two products.

[13]See, for example, Piskorski et al. (2010), Agarwal et al. (2011), Ghent (2011), Adelino et al. (2013), Mayer et al. (2014), and Ambrose et al. (2016).

[14]Third liens are relatively rare in practice.

We solve this problem by dynamic programming.[15] When a homeowner takes out a second lien, the homeowner sets the property's combined LTV to ℓ_0 and enters the next regime, regime 0, with no second lien opportunities remaining. Given the model's scaling feature, to ease computation and without loss of any generality, we normalize the property's service flow δ to one at the beginning of each regime. The homeowner has the option to default in each regime.

To fix matters, assume the homeowner retains his second lien option. We can take as given the previously obtained first lien. We also take as given the default and extraction triggers, δ_{B_1} and δ_F, as well as the total coupon payment rate c_1 in regime 1. Given the opportunity to take out a second lien, we follow our dynamic programming approach and begin with regime 0 in which the homeowner can no longer extract equity.

Recall, given the process for δ, the general solution, expression (1), to the ordinary differential equation governing valuation in our framework is

$$F(\delta) = f_1\delta^{x_1} + f_2\delta^{x_2} + \frac{a\delta}{r-\mu} + \frac{b}{r}.$$

Here $x_1 > 1$ and $x_2 < 0$ are the solutions to the quadratic equation

$$\frac{1}{2}\sigma^2 x(x-1) + \mu x - r = 0.$$

We determine f_1 and f_2 by relying on value matching conditions – two value matching conditions for each value function. The value function for the equity claim in the property is

$$E_0(\delta) = e_{01}\delta^{x_1} + e_{02}\delta^{x_2} + \frac{\delta}{r-\mu} - \frac{c_0}{r}, \tag{8}$$

[15]By way of notation, as before, a variable with a subscript denotes the variable's value when the subscripted number of equity extraction opportunities remain. When a variable is presented without a subscript this corresponds to the case where equity extraction is prohibited.

and the value function for debt is

$$D_0(\delta) = d_{01}\delta^{x_1} + d_{02}\delta^{x_2} + \frac{c_0}{r}. \tag{9}$$

Here c_0 denotes the coupon flow of the debt when there are no opportunities left to extract equity and $D_0(\delta)$ is the corresponding value of the *combined* first and second liens.[16]

In the case of no option left to extract equity (regime 0), the value matching conditions for the values of equity and debt needed to determine d_{01} and e_{01} are

$$\lim_{\delta\uparrow\infty} D_0(\delta) = \frac{c}{r}$$

$$\lim_{\delta\uparrow\infty}\left(E_0(\delta) - \frac{\delta}{r-\mu}\right) = -\frac{c}{r}.$$

That is, the risk of default is negligible when the service flow gets very high so the debt value becomes the value of receiving the coupon flow forever. Similarly the equity value is the value of receiving the service flow forever and paying the coupon flow forever. This implies $e_{01} = d_{01} = 0$ so that we can rewrite expressions (8) and (9) as

$$E_0(\delta) = e_{02}\delta^{x_2} + \frac{\delta}{r-\mu} - \frac{c_0}{r}, \tag{10}$$

and

$$D_0(\delta) = d_{02}\delta^{x_2} + \frac{c_0}{r}. \tag{11}$$

To determine d_{02} and e_{02}, we look at the value matching conditions at the corresponding

[16] It is more convenient to work with cumulative as opposed to individual mortgage loans for two reasons. First, the homeowner takes out a second lien to achieve a *cumulative* LTV ratio of ℓ_0. Second, the homeowner only cares about the total coupon payments on the *cumulative* mortgage loans when deciding whether or not to default.

foreclosure trigger, δ_{B_0}:

$$D_0(\delta_{B_0}) = (1-\alpha)A_1\frac{\delta_{B_0}}{\delta_0} \tag{12}$$

$$E_0(\delta_{B_0}) = 0. \tag{13}$$

Here A_1 is the value of the property financed by the new homeowner with an LTV of ℓ_1 with one extraction option remaining, δ_0 is the service flow of the property when initially bought, and α denotes the foreclosure costs. Plugging expressions (11) and (10) into expressions (12) and (13) yields

$$(1-\alpha)A_1\frac{\delta_{B_0}}{\delta_0} = d_{02}\delta_{B_0}^{x_2} + \frac{c_0}{r} \tag{14}$$

and

$$0 = e_{02}\delta_{B_0}^{x_2} + \frac{\delta_{B_0}}{r-\mu} - \frac{c_0}{r}. \tag{15}$$

The trigger for when the homeowner decides to cease paying the coupon flow to the lender, which then immediately triggers foreclosure, is determined by the smooth pasting condition

$$E_0'(\delta_{B_0}) = 0. \tag{16}$$

Expressions (14), (15), and (16) determine δ_{B_0}, e_{02}, and d_{02} for a *given* coupon flow of the debt c_0 and value of the property, A_1.

The coupon flow of the debt is determined when the homeowner extracts equity at δ_F which for now we take as given but will determine optimally later. Here the homeowner wants to lever up to an LTV of ℓ_0. That is, c_0 is determined as the solution to

$$\frac{D_0(\delta_F)}{D_0(\delta_F) + E_0(\delta_F)} = \ell_0.$$

Having solved the model with no extraction options left, regime 0, we now turn our attention to solving the model when there is one extraction option left. In regime 1 the coupon flow c_1 is lower and therefore the value functions of equity and debt, $E_1(\delta)$ and $D_1(\delta)$, respectively, will be different. In particular, we have

$$E_1(\delta) = e_{11}\delta^{x_1} + e_{12}\delta^{x_2} + \frac{\delta}{r-\mu} - \frac{c_1}{r},$$

and

$$D_1(\delta) = d_{11}\delta^{x_1} + d_{12}\delta^{x_2} + \frac{c_1}{r}.$$

To determine the four constants e_{11}, e_{12}, d_{11} and d_{12}, we need four value matching conditions: two value matching conditions, one for debt and one for equity, at the foreclosure trigger, δ_{B_1}, and two more at the extraction trigger, δ_F.

At the foreclosure trigger, δ_{B_1}, similar to the regime 0 case, we have

$$D_1(\delta_{B_1}) = (1-\alpha)A_1\frac{\delta_{B_1}}{\delta_0}$$
$$E_1(\delta_{B_1}) = 0.$$

In order to determine the trigger δ_{B_1}, we use the smooth pasting condition

$$E_1'(\delta_{B_1}) = 0.$$

At the extraction trigger, δ_F, the homeowner takes out the second lien. Thereafter, she pays the coupon flow c_0 but receives the proceeds from the new loan. That is,

$$E_1(\delta_F) = E_0(\delta_F) + (D_0(\delta_F) - D_{10}(\delta_F))$$

where $E_0(\delta_F)$ is the already derived value of equity in regime 0, $D_0(\delta_F)$ is the already derived value of *all* the outstanding debt in regime 0 and $D_{10}(\delta_F)$ is the value of the first lien subsequent to second lien's issuance. Given the coupon flow c_0, part of this flow, c_1, to be optimally determined, goes to the first lien holder, whose value is denoted $D_1(\delta)$ in regime 1, while the remainder of the flow, $c_0 - c_1$, goes to the second lien holder, that is, the *additional* debt that is issued at the time of equity extraction.

Notice that we value the second lien residually as the difference between the value of *all* the outstanding debt after the refinancing, with coupon flow c_0, and the value of the senior debt, with coupon flow $c_1 < c_0$. This is convenient for two reasons: (i) the homeowner only cares about the total coupon flow to all debt when she determines when to cease coupon payments, and (ii) the total debt value is independent of the sharing rule at the default trigger.

Because the homeowner has extracted equity via the additional loan, the default trigger δ_{B_0} is higher than δ_{B_1}. When default occurs at the trigger δ_{B_0}, we use the absolute priority rule to determine how the first and second lien holders share the foreclosure proceeds. This means that the first lien with coupon flow c_1 will receive $\min\{(1-\alpha)A_1\frac{\delta_{B_0}}{\delta_0}, D_1(\delta_0)\}$.

The value function for the first lien after the second lien has been issued, D_{10}, has the form

$$D_{10}(\delta) = d_{101}\delta^{x_1} + d_{102}\delta^{x_2} + \frac{c_1}{r}.$$

Now d_{101} is determined by the value matching condition

$$\lim_{\delta \uparrow \infty} D_{10}(\delta) = \frac{c_1}{r}$$

and d_{102} is determined by the value matching condition

$$D_{10}(\delta_{B_0}) = \min\{(1-\alpha)A_1 \frac{\delta_{B_0}}{\delta_0}, D_1(\delta_0)\}.$$

The latter condition means that potentially for some parameter values the right hand side of this value matching condition will be $(1-\alpha)A_1 \frac{\delta_{B_0}}{\delta_0}$ indicating that the first lien is still risky after refinancing and so the second lien holder will receive nothing in default, whereas for other parameter values, the first lien will be paid off at par after refinancing and there will be some value remaining for the second lien holder in case of default.

Finally, we can now determine the value matching condition for debt at the extraction trigger, δ_F:

$$D_1(\delta_F) = D_{10}(\delta_F).$$

The smooth pasting condition determining the trigger for equity extraction, δ_F, i.e., when the property owner finds it optimal to exercise her one and only extraction option is

$$E_1'(\delta_F) = \frac{d}{d\delta_F}\bigg(E_0(\delta_F) + (D_0(\delta_F) - D_{10}(\delta_F))\bigg).$$

Recall that c_0 will also be a function of δ_F, since as the homeowner waits for a higher service flow level, the additional loan required to achieve the desired LTV of ℓ_0 will be larger. The larger loan amount implies that the coupon flow, c_0, also increases with δ_F. This, in turn, means that the corresponding foreclosure trigger, δ_{B_0}, is a function of δ_F.

All the calculations up to here take as *given* values of the coupon flow c_1 and A_1, the value of the property optimally financed with an LTV ratio of ℓ_1 when the service flow level is δ_0 and with one refinancing option. We now determine c_1 at date zero when we assume an initial service flow level of δ_0 as the solution to

$$\frac{D_1(\delta_0)}{D_1(\delta_0) + E_1(\delta_0)} = \ell_1$$

and we simultaneously determine

$$A_1 = D_1(\delta_0) + E_1(\delta_0).$$

2.3.1 Pricing Properties under the Dynamic Financing Policy with Fixed LTV

Table 2 summarizes the effects of equity extraction for our base case specification of underlying parameter values for the case $\ell_0 = \ell_1$. For comparison purposes, we also provide corresponding values for the static financing case previously analyzed in which second liens are prohibited. Given the low risk of default with our benchmark calibration, even when the homeowner can take out a second lien, the pricing properties change little.

However, we see that property values are slightly lower when homeowners can extract equity. For example, when equity extraction is prohibited, $A = \$33.28$. Permitting homeowners to extract equity now results in a property value of $A_1 = \$33.25$ when the property is acquired, all else being equal. Intuitively, property values are lower in the presence of equity extraction opportunities because the likelihood of future defaults increases and default involves deadweight costs. For example, while the value of an Arrow-Debreu security contingent on default in the absence of equity extraction is ADD = \$0.008, its value given an equity extraction opportunity increases to $\text{ADD}_1 = \$0.012$. The resultant increase in expected foreclosure costs is capitalized in property values.

If homeowners can extract equity, the equivalent fixed waiting time to default is slightly shorter as compared to when homeowners are prohibited from extracting equity. Given the opportunity to extract equity results in $\text{EFWT}_1 = 88.7$ years, while EFWT = 96.4 years in the absence of equity extraction. Intuitively, equity extraction increases the homeowner's mortgage indebtedness and so, all else being equal, triggers an earlier default. Similarly, the equivalent fixed waiting time to default increases after the extraction option has been used. For example, in our base case parameterization, $\text{EFWT}_1 = 88.7$ years while $\text{EFWT}_0 = 96.4$ years. The reason that default risk is higher in regime 1 before the borrower exercises her extraction option is that for a service flow beginning at $\delta = 1$, the borrower has two ways of defaulting, either at δ_{B_0} or δ_{B_1}. This effect does not exist after exercising the extraction option.

2.3.2 Pricing Properties under the Dynamic Financing Policy with Higher Cumulative LTV with Second Lien

Table 2 also explores the effects of equity extraction for the case $\ell_0 > \ell_1$. Unlike the previous case of $\ell_0 = \ell_1$, EFWT declines after equity extraction when $\ell_0 > \ell_1$ and the probability of default, as measured by ADD, increases. Importantly, even a modest increase of ℓ_0 to 90% increases the probability of default by almost an order of magnitude. In the base case in which we do not permit equity extraction, ADD = \$0.008. When we allow the homeowner to extract equity up to a 90% LTV, even prior to equity extraction, ADD rises to ADD_1 = \$0.049. The increase in the probability of default becomes even more pronounced as we further increase ℓ_0.

Note that the higher risk of default to the first lien arises despite homeowners not all extracting equity. In the case of $\ell_0 = 0.95$, the homeowner does not extract equity until the service flow hits \$1.15. Not surprisingly, higher values of ℓ_0 are associated with higher foreclosure triggers δ_{B_0} corresponding to earlier foreclosure. The second lien's spread above the risk free rate is still low at 34 basis points for $\ell_0 = 0.9$. However, the spread rises to 121 and 361 basis points for combined LTVs (CLTVs) of 95% and 98%, respectively.

The foreclosure trigger prior to equity extraction, δ_{B_1}, falls when the homeowner has the option to extract at a higher LTV. The reason is that by defaulting before extracting equity, the borrower terminates the equity extraction option. Because this option has value, and greater value the higher the CLTV is at the equity extraction point, the threshold at which the homeowner defaults rises. This is reminiscent of the "competing risks" view of refinancing and default (see, for example, Deng et al. (2000)).

Finally, the fall in the property value due to the deadweight costs of foreclosure becomes increasingly noticeable as the maximum CLTV increases. Without equity extraction, the property is worth A = \$33.28 at origination. For $\ell_0 = 0.95$, the property is worth only A_1 = \$31.89 at origination of the first lien and still lower, A_0 = \$31.65, at equity extraction.

2.3.3 Sensitivity Analysis

Table 3 summarizes the model's sensitivities to changes in its underlying parameters. When we increase the risk free rate of interest, r, the homeowner exercises her option to extract equity sooner. Focusing on the case in which $\ell_0 = 0.8$, the homeowner extracts equity at a service flow of $\delta_F = \$1.40$ for $r = 3\%$, but only at a service flow of $\delta_F = \$1.36$ for $r = 7\%$. For the case of $\ell_0 = 0.95$, the extraction boundary falls from \$1.17 to \$1.13 as the risk free rate increases from 3% to 7%. This finding is consistent with the property of American options written on dividend paying stocks that exercise occurs earlier as the interest rate increases.

The effect of the risk free rate on the default trigger depends on whether or not the CLTV increases at extraction. For the case of $\ell_1 = \ell_0 = 0.8$, the homeowner defaults at a lower service flow both before and after extraction for $r = 3\%$ than for $r = 7\%$. This finding is also consistent with the properties of American options. When the CLTV at extraction is higher than at the origination of the first lien, after extraction the borrower defaults at a higher service flow as the interest rate increases. However, prior to extraction, a lower service flow value is necessary for the borrower to default when the interest rate is 7% rather than 3%. The reason is that because a lower service flow value triggers foreclosure after extraction in regime 0, the relative value of waiting is higher as the interest rate increases.

The properties of American options also imply that when the service flow volatility increases, the homeowner sets trigger points consistent with waiting longer to extract equity and to default. For example, we see that for $\ell_0 = 0.8$, the service flow at which the homeowner extracts equity when $\sigma = 3\%$ is $\delta_F = \$1.35$, which increases to a trigger service flow of $\delta_F = \$1.43$ when $\sigma = 7\%$. Default, on the other hand, is triggered at a service flow of

$\delta_{B_1} = \$0.78$ for $\sigma = 3\%$ but falls to $\delta_{B_1} = \$0.73$ for $\sigma = 7\%$. As expected, the equivalent fixed waiting times to default are shorter in the presence of more volatile housing service flows.

The extraction boundary also rises as foreclosure costs rise. For example, for $\ell_0 = 0.8$, the homeowner extracts equity at a service flow of $\delta_F = \$1.30$ when $\alpha = 20\%$ but for foreclosure costs of $\alpha = 30\%$, equity extraction is triggered much later at a higher service flow of $\delta_F = \$1.47$. We also see that higher foreclosure costs result in lower property values because of the greater deadweight costs. Similarly, borrowers face higher interest rates as foreclosure costs rise.

3 "Silent Seconds" and the Unraveling of MBS

The bursting of the U.S. housing bubble saw the unraveling of many MBS. For example, Ghent et al. (2016) find that by summer 2013, over a third of *AAA* private label MBS originated between 1999 and 2007 were in default while more than 80% of private label MBS securities originally rated investment grade but below *AAA* were in default.

Some critics have argued that these large downgrades reflected the fact that credit rating agencies (CRAs) simply underestimated the severity of the U.S. housing market downturn which caused a sharp increase both in the level of defaults as well as in the correlation of defaults across homeowners. Others have suggested that CRAs were blind to the fact that first lien borrowers could subsequently obtain second liens and, as a result, ignored the consequences of equity extraction on the performance of MBS.[17] These so-called "silent seconds" increased the likelihood that a homeowner would default in the event of a downturn in house prices. Moreover, the fact that so many U.S. homeowners relied on second liens to extract equity from their homes during the run-up in house prices through 2006 meant that they were more likely to default *en masse* when house prices subsequently fell.

We now investigate the extent to which the unraveling of MBS in the aftermath of

[17] See, for example, the discussion in Lewis (2010), page 100.

the bursting of the U.S. housing bubble can be attributed to CRAs ignoring borrowers' potential to take out a second lien. We also shed light on the role that CRAs underestimating the severity of the U.S. housing downturn may have played in the subsequent downgrades experienced by MBS.

3.1 A Hypothetical Cash MBS

We consider an originator who only originates first lien mortgages. In particular, we assume, without loss of generality, that at date t the lender has originated 1,000 first lien mortgages. Consistent with our valuation framework, each mortgage is an infinite-maturity loan characterized by the base case LTV of $\ell = 80\%$ and foreclosure costs of $\alpha = 25\%$. Each underlying property's service flow is (instantaneously) log normally distributed with the base case (instantaneous) drift of $\mu = 2\%$ and volatility of $\sigma = 5\%$. To model correlation between the underlying properties, we split a property's service flow into two components: a common component shared across all properties and a property-specific or idiosyncratic component. The common component has 40% of the volatility of the idiosyncratic component. However, the common component comprises 60% of the process. This calibration is consistent with the common share of home price variance estimated by Goetzmann (1993). Finally, the risk free rate of interest is, as before, fixed at $r = 5\%$.

Simultaneously, at date t the loan originator deposits the 1,000 first lien mortgages in a trust and receives, in return, the prevailing value of the loans. Relying on this pool of first lien mortgages as collateral, the trust issues an MBS consisting of two interest-bearing certificates, one senior and the other mezzanine, together with a non-interest bearing residual claim on the mortgage pool's cash flows. The interest rate owed on the certificates is the risk free rate plus the certificate's expected loss rate. We assume the MBS has a maturity of 10 years.

MBS prioritize payments to their constituent securities. In our case, the first priority is interest payments to the senior certificate. The second priority is interest payments to the mezzanine certificate. These interest payments are paid currently. Next are principal payments to the senior certificate, followed by principal payments to the mezzanine certificate. Any remaining cash flows are then allocated to the residual certificate. Principal payments are paid on an accrued basis on the maturity date of the MBS. This payout convention is required because we assume infinite-maturity mortgages are backing a finite maturity MBS.

If a default occurs, we assume the underlying property is immediately sold in a foreclosure sale and the resultant sale proceeds, net of administrative costs, are deposited by the trust in a risk free rate bearing account.[18] Losses are allocated first to the residual class, then to the mezzanine certificate, and finally to the senior certificate.

At the maturity of the MBS, the trustee sells the first lien mortgages remaining in the pool at their prevailing market prices. The trustee uses these proceeds together with the liquidation of any accounts in the trust arising from previous foreclosures to make principal payments according to the priority structure of the MBS. The trust is then terminated.

3.2 Sizing MBS

Apart from subordination, we assume that the MBS has no other form of credit enhancement. Therefore the credit rating assigned to a particular certificate depends solely on the degree of protection afforded the certificate by other certificates subordinate to it. The more subordination provided a particular certificate, the smaller the certificate's expected losses and so the higher its credit rating. Prior to the foreclosure crisis, Moody's, for example, assigned ratings for both corporate bonds and structured products based on the "idealized expected loss rates" given in Table 4. We rely on these loss rates in determining the ratings assigned to the interest-bearing certificates of our hypothetical MBS. Our loss rates include both loss of principal and loss of interest although, given the waterfall we specify, the vast majority of the losses are lost principal. As in practice, the residual certificate is not rated.[19]

[18] We assume that the pooling and servicing agreement of the MBS does not require the replacement of any defaulted loan in the pool regardless of how soon the default occurs.

[19] Post-financial crisis, some of the rating agencies, including Moody's, have issued separate scales for rating structured finance securities such as MBS. See Cornaggia et al. (forthcoming) for a discussion of the

To attain a particular credit rating in our framework requires us to determine the size of a certificate's principal so that the desired level of expected losses can be achieved given the underlying collateral's risk characteristics. To do so, we first increase the fraction of the MBS principal allocated to the senior certificate until across all of our simulations of the underlying correlated collateral the resultant fraction experiences an average loss rate equal to that allowed by the senior certificate's desired rating, for example, *Aaa*. Given we have sized the senior certificate, we then proceed in a similar fashion to size the mezzanine certificate so that its fraction has an average loss rate across all of our simulations equaling that allowed by its desired rating, for example, *Baa*3. The remaining fraction of the MBS principal is then allocated to the residual certificate.[20]

3.3 Simulation Results

We first assume that the CRA is naïve meaning that when rating the MBS it does not allow for the possibility that first lien borrowers may subsequently extract equity. To emulate this naïve CRA, we simulate, through the maturity date of the MBS the correlated service flow processes underlying each first lien mortgage included in the pool. We assume that the loan originators price the first liens assuming that homeowners cannot extract equity. Relying on our static financing policy framework, in which homeowners cannot extract equity but optimally default, we then calculate the losses incurred across the pool for each simulation when homeowners actually can extract equity by going up to a 95% LTV. We repeat this simulation exercise 1,000,000 times and size the MBS so that the naïve CRA rates the senior certificate as *Aaa* and the mezzanine certificate as *Baa*3.

Table 5 shows that, for the assumed base case parameters, the senior certificate accounts for approximately 97% of the MBS principal while the mezzanine certificate's size is approxi-

challenges of using the same scales for rating across asset classes.

[20]While it is the financial institution issuing the security that officially sizes the certificates of an MBS, prior to the financial crisis, issuers frequently consulted with the CRAs regarding what deal features were necessary for a certain portion of the deal to receive particular ratings. Hereafter, we thus sometimes refer to the CRA as being the institution that sizes the tranches.

mately 3% with only a tiny residual. While equity extraction adversely affects both tranches, the mezzanine tranche is far more affected than the senior tranche. The true rating of the senior tranche with equity extraction is actually $A1$, four notches lower than it should be because the losses are 80 times what the CRAs tolerate for a Aaa security. The mezzanine tranche is almost completely wiped out, losing essentially all of its value, by failing to account for the homeowner's option to extract equity. Rather than being in the investment grade category of $Baa3$, the actual rating of the mezzanine tranche is eleven notches lower at C.

An alternative way of understanding the erroneous ratings is to resize the tranches under the assumption that the CRA is savvy and understands the borrower's option to extract equity. The bottom panel of Table 5 shows the size of the tranches if the CRA takes into account the borrower's option to extract equity. The Aaa tranche is only 94% of the deal and now has double the subordination, an additional three percentage points. The biggest difference, however, is for the mezzanine tranche. While under the naïve CRA it has nearly no subordination and is almost completely wiped out, it now gets 4 percentage points of subordination when the CRA is savvy and takes into account the extraction option. In essence, ignoring equity extraction makes the mezzanine tranche akin to a residual rather than an investment grade security.

The last panel of Table 5 shows the losses on the overall pool. These results are the most relevant when considering the magnitude of potential losses to the GSEs since they guarantee whole pools. While in our base case parameterization, losses on the pool are a mere 0.04%, allowing equity extraction increases losses to 2.05%.

Our calculations to now assume that the homeowners in the pool are confronted with a wide variety of house price paths across our 1,000,000 simulations. We can also determine the losses incurred by homeowners in the pool, and therefore the losses passed on to MBS investors, if house prices behaved similarly to the path that actual U.S. house prices followed. To do so, we measure U.S. home prices by the monthly $FHFA$ non-seasonally adjusted repeat sales index. For purposes of our subsequent analysis, we consider MBS issuance dates of 2004, 2005, 2006, and 2007. To investigate the performance of MBS over the actual path of U.S. home prices, we now restrict our attention to scenarios in which the common component of the house price process[21] follows the actual monthly $FHFA$ index beginning in January 2004, January 2005, January 2006, or January 2007, respectively.[22]

To do so, we calculate losses across these particular paths assuming that homeowners optimally default but cannot extract equity. This allows us to determine what losses the MBS would have incurred due solely to the adverse realization of U.S. house prices. In Table 5, we see that relative to their original ratings, the senior certificate would remain unaffected by the adverse path of home prices if homeowners could not extract equity. Figure 1 shows that the losses from equity extraction are far greater for both tranches than those that would result solely from the adverse realization of home prices actually realized. Absent equity extraction, in fact, mezzanine investors in the 2004 MBS would fare even better than the stated rating as the rating corresponding to the actual loss experience corresponds to a rating of $Aa2$. Mezzanine investors in 2005, 2006, and 2007 fare worse with 2007 being the worst year. However, once extraction is permitted, we see from Table 5 that the mezzanine tranche is wiped out for every origination year.

Finally, Table 6 investigates the sensitivity of the effects of equity extraction on the sizing of MBS tranches to changes in the assumed underlying parameters. As before, given a particular set of parameters, the naïve CRA sizes the MBS so that the senior certificate is Aaa rated and the mezzanine certificate is $Baa3$ rated. We then take the given MBS and recalculate each certificate's expected losses assuming that homeowners can extract equity as well as default.

Notice that compared to the base case, the size of the Aaa rated senior certificate de-

[21] While it is based on service flows, our model has the property that at time points when a house is for sale, the house's value is a scalar of service flow. Therefore house values have the same stochastic properties as service flows.

[22] For the last year of the simulation for the 2007 historical experience, we revert to our base case assumption for the common component since there is not historical home price data available for 2017 as of the writing of this paper.

creases as the riskiness of the underlying collateral increases. In other words, the naïve CRA requires more subordination for the senior certificate to achieve a *Aaa* rating when the collateral's risk increases. For example, for a service flow volatility of only 3%, all else being equal, the size of the *Aaa* rated senior certificate is almost 100% and there is no mezzanine tranche. When the service flow volatility rises to 7%, only 85% of the MBS' principal is rated *Aaa*. In addition, the naïve credit rating agency requires more subordination in order for the senior certificate to be *Aaa* rated if interest rates are high and when foreclosure costs are high.

When we calculate expected losses across all 1,000,000 simulated house price paths assuming homeowners optimally extract equity as well as optimally default, the smallest downgrades correspond to the case in which volatility is high or foreclosure costs are high. This follows because it is only in these cases that there is substantive cushioning from the residual tranche. The senior tranche fares poorly in the seemingly safe scenario of a 3% service flow volatility because in this case there is a minuscule residual and actually no mezzanine tranche to protect the senior tranche. Not surprisingly, the largest downgrades result when property owners rely on equity extraction to increase their CLTV to 98%. Here the senior certificate would be downgraded to the non-investment grade *Ba*2 while the mezzanine tranche is completely wiped out.

4 Summary and Conclusions

Given the prominent role played by junior liens in extracting homeowner equity during the recent run-up in U.S. house prices, this paper explores the implications of homeowners' option to extract equity on the pricing and properties of residential mortgages, both first liens as well as junior liens, and, in turn, structured financial products based on the first lien mortgages with junior liens behind them. We find that ignoring equity extraction is a sufficient condition to generate the magnitude of losses observed on senior MBS tranches during the financial crisis. By contrast, adverse realizations of home prices are not enough in and of themselves to impair senior tranches. Nevertheless, we find that mezzanine tranches are far more affected by the presence of silent seconds than senior tranches. These results suggest that the potential to take on a junior lien subsequent to the origination of a first lien should be taken into account when pricing first mortgages and, especially, when structuring MBS. Our work also raises the question of why, in contrast to most other forms of debt financing, first lien residential mortgagees do not restrict mortgagors' ability to obtain subordinate financing.

Appendix: The Model with n Extraction Options

In this Appendix we detail the corresponding initial conditions as well as value-matching and smooth-pasting conditions characterizing the property owner's optimal default and equity extraction decisions for the general case in which the owner has n extraction options.

Assume the owner is in regime j in which j of the original n cash-out refinancing options remain. This means that the owner has already cash-out refinanced at each of the previous regimes $i = j+1, \ldots, n$. At the beginning of regime j we have the initial conditions:

$$D_{jj}(1) = P_j$$
$$E_j(1) = A_j - P_j$$

where P_j denotes the cumulative principal borrowed after the owner's jth refinancing and A_j denotes the then prevailing value of the underlying property. The total coupon payment rate the owner will pay during regime j, denoted c_j, is determined so that

$$\frac{P_j}{A_j} = \ell.$$

The default value-matching and smooth-pasting conditions in regime j are given by

$$E_j(\delta_{B_j}) = 0$$
$$E'_j(\delta_{B_j}) = 0$$
$$D_{ij}(\delta_{B_j} \prod_{k=j+1}^{i} \delta_{F_k}) = \min\left\{(1-\alpha)A_n \delta_{B_j} \prod_{k=j+1}^{i} \delta_{F_k}, \frac{c_i}{r}\right\}$$

for $i = j, \ldots, n$. The homeowner defaults when the house's service flow is sufficiently low relative to the total coupon payment rate, c_j, to all the mortgage loans issued. In the event of default, the homeowner defaults on all mortgages and lenders are assumed to foreclose instantaneously thereafter and allocate the available proceeds amongst the existing liens according to absolute priority. To keep track of this, we have $n - j + 1$ value-matching conditions for the cumulative mortgage values. In particular, cumulatively all the mortgages issued in all regimes up to and including regime j, this value being denoted by D_{jj}, will receive $(1-\alpha)A_n\delta_{B_j}$ in case of default. This reflects the fact that the creditors receive the property value net of foreclosure costs, α, and that the property can be sold to a new homeowner who again will have exactly n refinancing options.

Similarly, for $j \geqslant 1$, the refinancing value-matching and smooth-pasting conditions in regime j are given by[23]

$$E_j(\delta_{F_j}) = \delta_{F_j} A_{j-1} - D_{j,j-1}(\delta_{F_j})$$
$$E'_j(\delta_{F_j}) = A_{j-1} - D'_{j,j-1}(\delta_{F_j})$$
$$D_{ij}(\delta_{F_j} \prod_{k=j+1}^{i} \delta_{F_k}) = D_{i,j-1}(\delta_{F_j} \prod_{k=j+1}^{i} \delta_{F_k})$$

for $i = j, \ldots, n$.

Since D_{ij} is the *cumulative* value of all the mortgages issued to the homeowner in regime i and all previous regimes (with higher indices, $i+1, \ldots, n$), we can determine the value (as of regime j) of *just* the mortgage issued in regime i by calculating

$$D_{ij}(\delta_i) - \frac{1}{\delta_{F_{i+1}}} D_{i+1,j}(\delta_{F_{i+1}}\delta_i)$$

for $i = 0, \ldots, n-1$ and $j = 0, \ldots, i$. Similarly, the coupon payment rate of the mortgage *just* issued in regime i is calculated as

$$c_i - \frac{c_{i+1}}{\delta_{F_{i+1}}},$$

for $i = 0, \ldots, n-1$.

[23]Note that for the case $j = 0$ there are no cash-out refinancing opportunities remaining and so these value-matching and smooth-pasting conditions do not apply.

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Table 1: Valuation Without Equity Extraction

This table provides values of the underlying property (A), first lien mortgage principal (P) and mortgage rate (y) in addition to the critical service flows (δ_B) at which the homeowner optimally defaults with corresponding equivalent fixed waiting time to default (EFWT) and values of Arrow-Debreu security contingent on default (ADD). We assume a base case of parameter values as well as perturbing the base case by assuming an alternative parameter value as indicated in the Table's column headings. The Base Case assumes that the risk free rate, r, is 0.05, the volatility of the property service flow is 5%, the foreclosure discount, α, is 25%, and the LTV at origination, ℓ is 80%. We normalize the property's service flow, δ, to one at origination.

		r		σ		α		ℓ	
	Base Case	3%	7%	3%	7%	20%	30%	70%	90%
A	\$ 33.28	\$ 99.81	\$ 19.97	\$ 33.33	\$ 33.01	\$ 33.29	\$ 33.27	\$ 33.33	\$ 32.83
P	\$ 26.62	\$ 79.85	\$ 15.98	\$ 26.67	\$ 26.41	\$ 26.63	\$ 26.62	\$ 23.33	\$ 22.98
y	5.01%	3.01%	7.01%	5.00%	5.09%	5.01%	5.01%	5.00%	6.56%
δ_B	\$ 0.757	\$ 0.755	\$ 0.759	\$ 0.783	\$ 0.728	\$ 0.757	\$ 0.757	\$ 0.662	\$ 0.855
ADD	\$ 0.008	\$ 0.010	\$ 0.007	\$ 0.000	\$ 0.051	\$ 0.008	\$ 0.008	\$ 0.001	\$ 0.067
EFWT	96	154	71	224	59	96	96	143	54

Table 2: Valuation with Equity Extraction: Base Case Parameterization

This table provides values of the underlying property (A), second lien mortgage rate (y) and cumulative mortgage rate ($\bar{y}$) when the property owner can optimally extract equity. We assume the base case of parameter values (r=0.05, $\mu = 0.02$, $\sigma = 0.15$, $\alpha = 0.25$, and $\ell_1 = 0.8$). The critical service flows at which the owner optimally extracts equity (δ_F) and optimally defaults (δ_B as a percentage of service flow at purchase or equity extraction) with corresponding equivalent fixed waiting times to default (EFWT) and values of Arrow-Debreu security contingent on default (ADD) are also provided. Regime 1 refers to the period prior to equity extraction and regime 0 refers to the period after equity extraction. We normalize the property's service flow, δ, to one at the beginning of each regime.

	A	P	y	$\bar{y}$	δ_B	δ_F	ADD	EFWT
No Equity Extraction Allowed								
	\$ 33.28	\$ 26.62	5.012%	5.012%	\$ 0.757		\$ 0.008	96.4
With Equity Extraction Option and $\ell_0 = \ell_1 = 0.8$								
Regime 1	\$ 33.25	\$ 26.60	5.012%	5.012%	\$ 0.757	\$ 1.38	\$ 0.012	88.7
Regime 0	\$ 33.28	\$ 26.63	5.011%	5.012%	\$ 0.757		\$ 0.008	96.4
With Equity Extraction Option and $\ell_0 = 0.9, \ell_1 = 0.8$								
Regime 1	\$ 32.90	\$ 26.32	5.016%	5.016%	\$ 0.755	\$ 1.22	\$ 0.049	60.1
Regime 0	\$ 32.84	\$ 29.55	5.335%	5.102%	\$ 0.855		\$ 0.067	54.1
With Equity Extraction Option and $\ell_0 = 0.95, \ell_1 = 0.8$								
Regime 1	\$ 31.89	\$ 25.51	5.028%	5.028%	\$ 0.749	\$ 1.15	\$ 0.149	38.1
Regime 0	\$ 31.65	\$ 30.07	6.213%	5.337%	\$ 0.910		\$ 0.196	32.5
With Equity Extraction Option and $\ell_0 = 0.98, \ell_1 = 0.8$								
Regime 1	\$ 29.35	\$ 23.48	5.063%	5.063%	\$ 0.732	\$ 1.10	\$ 0.338	21.7
Regime 0	\$ 28.85	\$ 28.28	8.606%	5.927%	\$ 0.951		\$ 0.416	17.5

Table 3: Valuation with Equity Extraction: Comparative Statics

		A	P	y	$\bar{y}$	δ_B	δ_F	ADD	EFWT
					r=3%				
$\ell_1 = 0.8$	Regime 1	\$ 99.66	\$ 79.72	3.009%	3.009%	\$0.755	\$ 1.40	\$ 0.016	139
	Regime 0	\$ 99.81	\$ 79.85	3.009%	3.009%	\$0.755		\$ 0.010	154
$\ell_0 = 0.95$	Regime 1	\$ 94.75	\$ 75.80	3.013%	3.013%	\$0.751	\$ 1.17	\$ 0.174	58
	Regime 0	\$ 94.53	\$ 89.80	3.767%	3.221%	\$0.909		\$ 0.208	52
					r=7%				
$\ell_1 = 0.8$	Regime 1	\$ 19.96	\$ 15.97	7.014%	7.014%	\$0.758	\$ 1.36	\$ 0.009	67
	Regime 0	\$ 19.97	\$ 15.98	7.012%	7.014%	\$0.759		\$ 0.007	71
$\ell_0 = 0.95$	Regime 1	\$ 19.25	\$ 15.40	7.043%	7.043%	\$0.749	\$ 1.13	\$ 0.131	29
	Regime 0	\$ 19.05	\$ 18.10	8.624%	7.438%	\$0.911		\$ 0.187	24
					$\sigma = 3\%$				
$\ell_1 = 0.8$	Regime 1	\$ 33.33	\$ 26.67	5.000%	5.000%	\$0.783	\$ 1.35	\$ 0.000	217
	Regime 0	\$ 33.33	\$ 26.67	5.000%	5.000%	\$0.783		\$ 0.000	224
$\ell_0 = 0.95$	Regime 1	\$ 33.08	\$ 26.47	5.001%	5.001%	\$0.781	\$ 1.13	\$ 0.028	72
	Regime 0	\$ 33.04	\$ 31.38	5.196%	5.051%	\$0.931		\$ 0.037	66
					$\sigma = 7\%$				
$\ell_1 = 0.8$	Regime 1	\$ 32.81	\$ 26.25	5.089%	5.089%	\$0.727	\$ 1.43	\$ 0.073	52
	Regime 0	\$ 33.01	\$ 26.40	5.084%	5.087%	\$0.728		\$ 0.052	59
$\ell_0 = 0.95$	Regime 1	\$ 30.41	\$ 24.33	5.123%	5.123%	\$0.714	\$ 1.17	\$ 0.278	26
	Regime 0	\$ 30.10	\$ 28.59	7.471%	5.758%	\$0.892		\$ 0.345	21
					$\alpha = 20\%$				
$\ell_1 = 0.8$	Regime 1	\$ 33.26	\$ 26.61	5.010%	5.010%	\$0.757	\$ 1.30	\$ 0.012	88
	Regime 0	\$ 33.29	\$ 26.63	5.009%	5.010%	\$0.757		\$ 0.008	96
$\ell_0 = 0.95$	Regime 1	\$ 32.08	\$ 25.67	5.023%	5.023%	\$0.750	\$ 1.08	\$ 0.167	36
	Regime 0	\$ 31.98	\$ 30.38	6.190%	5.279%	\$0.910		\$ 0.194	33
					$\alpha = 30\%$				
$\ell_1 = 0.8$	Regime 1	\$ 33.24	\$ 26.59	5.014%	5.014%	\$0.757	\$ 1.47	\$ 0.011	90
	Regime 0	\$ 33.27	\$ 26.62	5.013%	5.014%	\$0.757		\$ 0.008	96
$\ell_0 = 0.95$	Regime 1	\$ 31.74	\$ 25.39	5.032%	5.032%	\$0.748	\$ 1.22	\$ 0.131	41
	Regime 0	\$ 31.32	\$ 29.76	6.241%	5.396%	\$0.911		\$ 0.199	32

Table 4: Moody's Ratings for Corporate Bonds and Their Expected Loss Criteria

This table shows Moody's ratings for corporate bonds and their corresponding expected loss rates. Expected loss rates are over a four-year horizon.

Corporate Rating	Expected Loss Rate
Aaa	0.0010%
Aa1	0.0116%
Aa2	0.0259%
Aa3	0.0556%
A1	0.1040%
A2	0.1898%
A3	0.2870%
Baa1	0.4565%
Baa2	0.6600%
Baa3	1.3090%
Ba1	2.3100%
Ba2	3.7400%
Ba3	5.3845%
B1	7.6175%
B2	9.9715%
B3	13.2220%
Caa1	17.8634%
Caa2	24.1340%
Caa3	36.4331%
Ca	50.0000%
C	80.0000%
D	90.0000%

Table 5: MBS and "Silent Second": Base Case Parameters

We size a cash MBS and determine its certificates' expected losses under a variety of assumptions. We assume the base case of parameter values. In Panel A, in the Baseline case, a naïve credit rating agency relies on 1,000,000 simulation paths in which homeowners optimally default but cannot extract equity. Originators price the first liens assuming property owners cannot extract equity. In the "U.S. Experience", the common component of home prices follows the behavior of U.S. house prices since origination. Loss rates are normalized to correspond to a four year horizon shown in Table 4. Panel B shows the correct sizing of the tranches when the credit rating agency takes into account the borrowers' ability to extract equity.

Panel A: Naïve Credit Rating Agency

	Baseline		All Paths: Extraction Permitted		U.S. Experience: No Extraction		U.S. Experience: Extraction Permitted	
Size	Loss Rate	Rating	Loss Rate	Rating	Loss Rate	Rating	Loss Rate	Rating
2004 Originations:								
0.968	0.001%	Aaa	0.080%	A1	0.000%	Aaa	0.975%	Baa3
0.031	1.309%	Baa3	64.831%	C	0.014%	Aa2	99.997%	D
2005 Originations:								
0.968	0.001%	Aaa	0.080%	A1	0.000%	Aaa	0.695%	Baa3
0.031	1.309%	Baa3	64.831%	C	2.459%	Ba2	99.997%	D
2006 Originations:								
0.968	0.001%	Aaa	0.080%	A1	0.000%	Aaa	0.276%	A3
0.031	1.309%	Baa3	64.831%	C	15.026%	Caa1	99.822%	D
2007 Originations:								
0.968	0.001%	Aaa	0.080%	A1	0.000%	Aaa	0.233%	A3
0.031	1.309%	Baa3	64.831%	C	17.958%	Caa2	98.383%	D

Panel B: Saavy Credit Rating Agency (Correct Sizing if Extraction Permitted)

	All Paths: No Extraction		All Paths: Extraction Permitted	
Size	Loss Rate	Rating	Loss Rate	Rating
0.941	0.000%	Aaa	0.001%	Aaa
0.021	0.020%	Aa2	1.309%	Baa3

Panel C: Total Losses on Pool

All Paths: Baseline	All Paths: Extraction Permitted
0.042%	2.045%

Table 6: MBS and "Silent Seconds": Comparative Statics

We size a cash MBS and determine its certificates' expected losses under a variety of assumptions. We perturb the base case of parameter values by assuming an alternative parameter value as indicated. A naïve credit rating agency relies on 1,000,000 simulation paths in which homeowners optimally default but cannot extract equity. Originators price the first liens assuming property owners cannot extract equity. Loss rates are normalized to correspond to the four year horizon shown in Table 4.

	Baseline		All Paths with Extraction Permitted	
Size	Loss Rate	Rating	Loss Rate	Rating
		$r = 3\%$		
0.970	0.001%	Aaa	0.012%	Aa2
0.030	1.309%	Baa3	37.068%	Ca
		$r = 7\%$		
0.964	0.001%	Aaa	0.286%	A3
0.034	1.309%	Baa3	89.451%	D
		$\sigma = 3\%$		
0.999	0.001%	Aaa	0.202%	A3
—	1.309%	Baa3	—%	—
		$\sigma = 7\%$		
0.847	0.001%	Aaa	0.005%	Aa1
0.138	1.309%	Baa3	14.922%	Caa1
		$\alpha = 20\%$		
0.974	0.001%	Aaa	0.157%	A2
0.025	1.309%	Baa3	75.759%	C
		$\alpha = 30\%$		
0.960	0.001%	Aaa	0.020%	Aa2
0.038	1.309%	Baa3	51.33%	C
		$\ell_0 = 80\%$		
0.968	0.001%	Aaa	0.00103%	Aa1
0.031	1.309%	Baa3	1.140%	Baa3
		$\ell_0 = 90\%$		
0.968	0.001%	Aaa	0.003%	Aa1
0.031	1.309%	Baa3	7.498%	B1
		$\ell_0 = 98\%$		
0.968	0.001%	Aaa	3.365%	Ba2
0.031	1.309%	Baa3	99.997%	Ca

Figure 1: Losses from Equity Extraction

The first two bars of each figure (corresponding to "All Simulated") show the losses, normalized to correspond to a 4-year horizon for all simulated paths. We size the *Aaa* tranche to have a loss of 0.001% under the assumption of no extraction. We size the *Baa*3 tranche to have a loss of 1.3%. These losses are those shown in the first bar. The last 8 bars show the losses without and with the equity extraction option when the common component of home prices follows the FHFA national index from January of the year of origination shown.