Appendix for "Infrequent Housing Adjustment, Limited Participation, and Monetary Policy"

Andra Ghent[1]

This Draft: October 28th, 2011

Abstract

This paper asks why monetary contractions have strong effects on the housing market. The paper presents a model with staggered housing adjustment in which monetary policy has real effects in the absence of any rigidity in producer pricing or wages. Limited participation in financial markets leads to a rise in the real mortgage rate following an increase in the nominal short rate. Since households must take on a mortgage to consume housing, the rise in the real interest rate reduces the share of residential investment in output.

JEL: E52, G14, R21, R31.

Key Words: Sticky Housing. Residential Investment. Limited Participation. Monetary Policy. Mortgages.

[1] Department of Real Estate, Baruch College, Zicklin School of Business. Email: andra.ghent@baruch.cuny.edu, Phone: (646)660-6929.

1 Solving the Model

The budget constraint and FOCs for housing, cash, and labor supply for a type 0 household are

$$C_{t,0} = L_{t,0} + \frac{M_{t-1,J-1}}{P_t} - \frac{M_{t,0}}{P_t} - F_t(H_{t,0} - (1-\delta)\, H_{t-J,0}) \tag{1}$$

$$\begin{aligned} & E_t \sum_{j=0}^{J-1} \beta^j u_2\left(C_{t+j,j}, H_{t,0}\right) + E_t \beta^J F_{t+J}\left(1-\delta\right) u_1\left(C_{t+J,0}, H_{t+J,0}\right) \\ &= E_t \sum_{j=0}^{J-1} \beta^j u_1\left(C_{t+j,j}, H_{t,0}\right) F_t. \end{aligned} \tag{2}$$

$$u_1\left(C_{t,0}, H_{t,0}\right) = \omega_1^h \frac{P_t}{M_{t,0}} + \beta E_t \left[u_1\left(C_{t+1,1}, H_{t,0}\right) \frac{P_t}{P_{t+1}} \right] \tag{3}$$

$$u_1\left(C_{t,0}, H_{t,0}\right) = \omega_2 \left(1 - L_{t,0}\right)^{-\eta_L}. \tag{4}$$

The budget constraint and FOCs for cash and labor supply for a type $j \neq 0, J-1$ are

$$C_{t,j} = L_{t,j} + \frac{M_{t-1,j-1}}{P_t} - \frac{M_{t,j}}{P_t} - F_{t-j}\left(H_{t-j,0} - (1-\delta)\, H_{t-j-J,0}\right) \tag{5}$$

$$u_1\left(C_{t,j}, H_{t-j,0}\right) = \omega_1^h \frac{P_t}{M_{t,j}} + \beta E_t \left[u_1\left(C_{t+1,j+1}, H_{t-j,0}\right) \frac{P_t}{P_{t+1}} \right] \tag{6}$$

$$u_1\left(C_{t,j}, H_{t-j,0}\right) = \omega_2 \left(1 - L_{t,j}\right)^{-\eta_L}. \tag{7}$$

and for a type $j = J-1$ household they are

$$C_{t,J-1} = L_{t,J-1} + \frac{M_{t-1,J-2}}{P_t} - \frac{M_{t,J-1}}{P_t} - F_{t-(J-1)}\left(\right) \tag{8}$$

$$u_1\left(C_{t,J-1}, H_{t-(J-1),0}\right) = \omega_1^h \frac{P_t}{M_{t,J-1}} + \beta E_t \left[u_1\left(C_{t+1,0}, H_{t+1,0}\right) \frac{P_t}{P_{t+1}} \right] \tag{9}$$

$$u_1\left(C_{t,J-1}, H_{t,_{t-(J-1),0}}\right) = \omega_2 \left(1 - L_{t,J-1}\right)^{-\eta_L}. \tag{10}$$

The first order conditions for a lender are

$$u'(C_{t,l}) = \omega_1^l \frac{P_t}{M_{t,l}} + \beta E_t \left\{ u'(C_{t+1,l}) \frac{P_t}{P_{t+1}} \right\} \tag{11}$$

$$u'(C_{t,l}) = E_t \left\{ \sum_{j=0}^{J-1} \beta^j u'(C_{t+j,l}) F_t \right\} \tag{12}$$

$$u'(C_{t,l}) = E_t \left\{ u'(C_{t+1,l}) R_{t,SHORT} \frac{P_t}{P_{t+1}} \right\} \tag{13}$$

We also have the first-order accurate market-clearing conditions

$$L_t = \frac{\epsilon}{J} \sum_{j=0}^{J-1} L_{t,j} \tag{14}$$

$$C_t = \frac{\epsilon}{J} \sum_{j=0}^{J-1} C_{t,j} + (1-\epsilon) C_{t,l} \tag{15}$$

$$M_t = \frac{\epsilon}{J} \sum_{j=0}^{J-1} M_{t,j} + (1-\epsilon) M_{t,l} \tag{16}$$

$$L_t = C_t + \frac{\epsilon}{J} \left(H_{t,0} - (1-\delta) H_{t-J,0} \right) \tag{17}$$

The monetary authority follows

$$\frac{R_{t,SHORT}}{R_{SHORT}} = \left(\frac{R_{t-1,SHORT}}{R_{SHORT}} \right)^{\rho_M} \exp\left(\varepsilon_t^M\right) \tag{18}$$

by adjusting the money supply according to

$$M_t = M_{t-1} + R_{t-1,SHORT} b_{t-1,GOV} - b_{t,GOV}. \tag{19}$$

This constitutes a system of $J*3+10$ difference equations in the $J*3+10$ unknowns $\{C_j, m_j, L_j, j = 0, ..., J-1\}$, C_l, M_l, C, H_0, Y, P, F, R_{SHORT}, b_{GOV}, and M.

2 The Steady State

The Euler equations for labor supply for all home buyers at an interior solution evaluated at the steady state imply

$$u_1(C_j, H_0) = \omega_2 (1 - L_j)^{-\eta_L}, \; j = 0, .., J-1$$

where C_j, H_0, and L_j, are the steady state values of consumption, housing, and hours worked. This implies that their marginal utility of nondurables is the same conditional on the housing stock. Since only the type 0 household chooses housing, nondurables must be identical for all j. The Euler equations for real balances for all households j,

$$u_1(C_j, H_0)(1-\beta) = \omega_1^h m_j^{-1},\ j = 0, ..., J-1$$

then imply m_j is the same for all home buyers.

To calibrate steady state real balances, I assume that $m_j = \frac{M_j}{P} = \frac{C_j}{\nu}$ and $m_l = \frac{M_l}{P} = \frac{C_l}{\nu}$ where ν is velocity. We can then solve for the steady state values of C_j, C_s, H and the parameter γ by evaluating (7), the budget constraints for a lender and a home buyer, and the resource constraint at the steady state. We have

$$\begin{aligned}
C_j &= Cshare\left(L_j - \frac{C_j}{\nu}\left(1-\frac{1}{\pi}\right)\right) \\
&= CshareL_j\left(1+\frac{Cshare}{\nu}\left(1-\frac{1}{\pi}\right)\right)^{-1} \\
H &= \frac{L_j - C_j\left(1+\frac{1}{\nu}\left(1-\frac{1}{\pi}\right)\right)}{\delta F} \\
F &= \left(\sum_{j=0}^{J-1}\beta^j\right)^{-1} \\
C_s &= \left(L - \epsilon C_j - \frac{\epsilon}{J}\delta H\right)/(1-\epsilon) \\
\gamma &= \left(\frac{C_j}{H}\right)^{\theta-1} F\left(1-\beta^J F(1-\delta)\right)
\end{aligned}$$

The steady short rate, real money supply, and real stock of government bonds are then

$$\begin{aligned}
R_{SHORT} &= \frac{\pi}{\beta} \\
m &= \epsilon m_j + (1-\epsilon) m_s \\
\frac{b_{GOV}}{P} &= -m\frac{\left(1-\frac{1}{\pi}\right)}{\left(1-\frac{1}{\beta}\right)}
\end{aligned}$$

I use the DYNARE software suite to solve a first-order log linearization of the model. DYNARE uses a generalized Schur decomposition to solve the model: see Klein (2000) and Sims (2001) for additional details on this general approach to solving systems of linear difference equations.

References

Klein, P., 2000. "Using the Generalized Schur Form to Solve a Multivariate Linear Rational Expectations Model." Journal of Economic Dynamics & Control 24, 1405-1423.

Sims, C.A., 2001. "Solving Linear Rational Expectations Models." Computational Economics 20, 1-20.